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THE EFFECT OF RADOME SCATTERING ON

ECM ANTENNA PATTERNS

THESIS

AFIT/GE/EE/81D-52

Robert K. Schneider
2Lt USAF

Prepared as partial fulfillment of the requirements for the Degree of Master of Science

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THE EFFECT OF RADOME SCATTERING ON ECM ANTENNA PATTERNS

THESIS

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of the Air Force Institute of Technology

Air University

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by

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Preface

This thesis pertains to a very significant problem, the formation of secondary lobes in antenna radiation patterns due to dielectric radomes. The problem is not understood well enough to even suggest a sure approach to obtaining a solution. This study is only one of several studies which could be made. Yet, it is meant to be a foundation which other studies could be built upon to eventually engineer a definite solution. Several interesting complexities of the problem are realized by this study and are discussed to set that foundation to work from.

The symbols used throughout this paper are the same as those used in the literature sighted or are defined as they occur. The primary reference used was Time-Harmonic Electromagnetic Fields by R. F. Harrington (Ref. 2), and the symbols follow almost without variation from that text.

It should be pointed out that this thesis contains theory on series solution and moment method solution formulations. But, most of the time for this study was spent learning about and working on the Hewlett-Packard 21MX M series minicomputer. This system was chosen to be used because of its easy accessability. A sincere thanks is extended to ASD/ENAMA for the unlimited use of their system as well as for the use of other resources and also to the support from that branch especially through William J. Kent, electronic warfare engineer.

An error was discovered having to do with the results from the series solution presented herein. The theory is correct, but the calculations of the coefficient F_n is double what it should be for orders not equal to zero. This error should not change the results for the scatterer of six

tenths wavelength outside diameter where the zero order term should dominate. But, for a diameter of sixty wavelengths the error could become significant and should be checked against the results in Appendix E. The results from the moment method are considered to be correct.

A special thanks goes out to my advisor, Major August Golden, Jr.. He provided the motivation and guidance which was needed throughout this study. And, the scrutinized reading of this thesis by Bill Kent and Captain Thomas Johnson was also sincerely appreciated.

Robert K. Schneider

Dedication

This thesis is dedicated to my wife, Linda, and to my family at home. They all had confidence that the hard work would pay off and were the inspiration I needed.

Table of Contents

List of Figures

Figure		Page

List of Tables

Abstract

The problem of scattering by thin cylindrical dielectric shells of large circular cross sections is approached by two methods: (1) an infinite series of eigenfunctions, and (2) the method of moments. Numerical results are presented for shell radii of 0.3λ, 3.0λ, and 30λ, the source being an electric line current near but external to the shell. Computer programs are presented which implement these two solutions. When the scattering structure does become large limitations on numerical results are encountered due to computer memory and speed limitations. Other difficulties are also encountered in an analysis of such scattering problems and are presented and discussed along with recommendations to resolve such difficulties.

I. Introduction

Scattering by perfectly conducting obstacles is exhaustively discussed in the literature (Ref. 1; 2; 3; 4; and 5). The same is not true for scattering by dielectric bodies. Several studies have been done dealing with scattering from planar dielectric surfaces (Ref. 1; 2; 3; 4; 5; 6; and 7). Pathak (Ref. 7) gives an exhaustive analysis of the diffraction of a TM_0. Surface wave by a dielectric slab terminated and flush mounted in a perfectly conducting surface. This thesis is motivated by the study of radomes which are usually curved surfaces. Studies of the scattering by curved dielectric surfaces have been rather limited. Most of the articles found in the literature concentrate on scattering by solid dielectric cylinders of circular or elliptical cross section (Ref. 8; 9; 10; 11; 12). Some studies have been done concerning other curved structures (Ref. 13 to 20).

Kuester and Chang (Ref. 14) investigate the continuous radiation of a wave as it travels along a uniformly curved section of open waveguide. They present a technique for determining the radiation loss at a discontinuity in curvature. A very modest study of the radome scattering problem has been done modeling with a cylindrical shell. Lewin, Chang, and Kuester (Ref. 13:95) examine the case of plane wave incidence on the scattering surface, which lends insight to the problem but is not exactly the case when the source is assumed to be very close to the scattering surface.

Thiele (Ref. 21:306) explains the theory behind the method of moments as do Harrington (Ref. 22) and Richmond (Ref. 23). Harrington and Richmond apply this method directly to scattering from a dielectric, circular cylindrical shell for small geometries; that is, cross sections of less than

$40/\pi$ wavelengths. Richmond details the technique of setting up the equations to calculate the scattered field and discusses employing the Lagrange Interpolation method (Ref. 24) to reduce the size of the matrix arrived at. Richmond's study only applies to geometrics much smaller than the sixty wavelength cross section (see Appendix A) of concern in this report. Also, Harrington (Ref. 2:198) develops the basics for setting up a series (or eigenfunction) solution for this problem using cylindrical wave functions.

This thesis develops an infinite series of eigenfunctions as a solution for the scattering of electromagnetic waves incident from a nearby electric current filament upon a dielectric, cylindrical shell of circular cross section. A computer program is used to generate numerical results from this series solution. These results are compared to Richmond's (Ref. 23:338) for the geometry indicated there. The solution by the method of moments is also developed and programmed. Results are compared to those from the infinite series solution. The geometry is then enlarged to approach the sixty wavelength cross section of interest. The objective is to compare the results obtained from the infinite series solution and those from the moment method solution, and thus obtain confidence in the solution. Finally, difficulties encountered from either solution method are enumerated and discussed.

II. The Problem and Approach

The radome and antennas configuration to be modeled is shown in Figure 1. The antennas are located six inches behind the radome in the metallic fuselage and have beam maximum at 45° from the skin of the aircraft. The resulting antenna radiation pattern is shown in Figure 2.

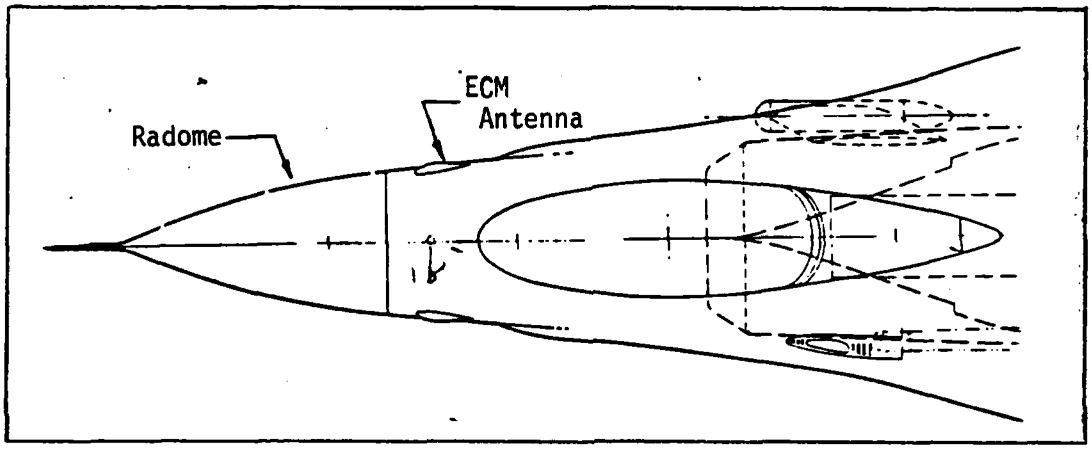

Figure 1. Antenna/Radome configuration.

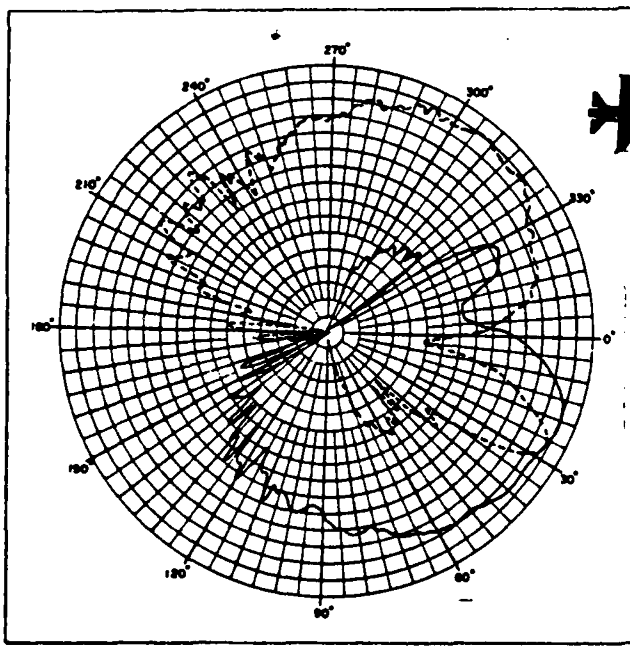

Figure 2. Present radiation pattern.

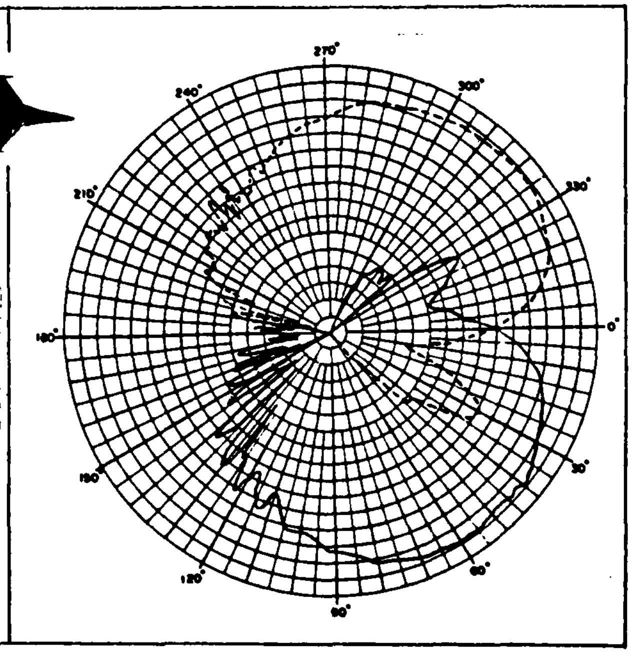

Figure 3. Recommended radiation pattern.

Note in Figure 2 the secondary lobes at plus and minus thirty degrees, their magnitude being only about 1dB down from the primary lobe. The fact that the patterns overlap off the zero degree line is likely due to asymmetry in product manufacturing and installation of the aircraft antennas and radome. The recommended radiation pattern appears in Figure 3. The secondary lobes have been decreased to about 6dB below the primary lobe and cross over occurs at zero degrees. A comparison of the ideal and typical patterns radiated appears in Figure 4.

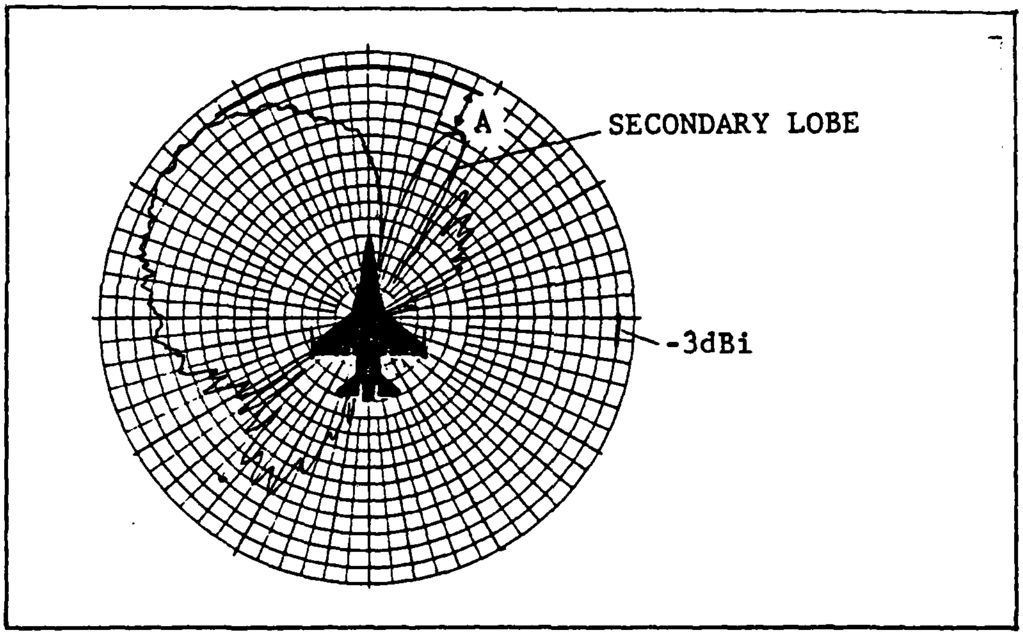

Figure 4. Formation of the secondary lobe from the left-hand antenna relative to the ideal pattern.

The main concern here is the cause of the undesired secondary lobe. This thesis does not answer that question but investigates an approach which might be useful in engineering an answer. As a first step toward that answer, the problem is modeled as shown in Figure 5.

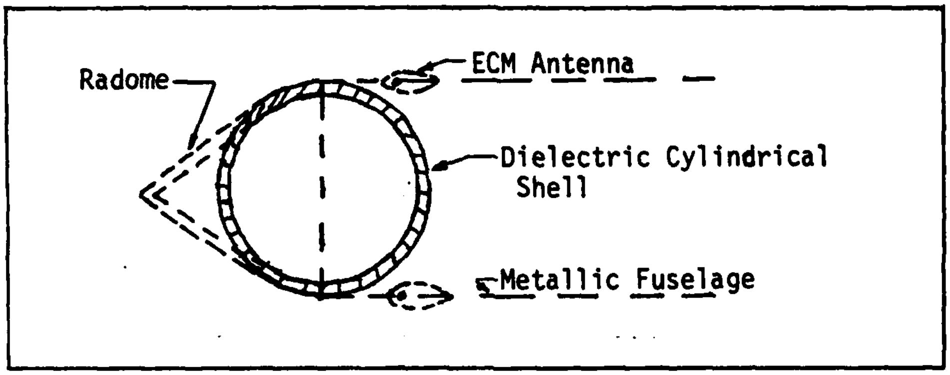

Figure 5. Modeling the radome as a dielectric cylindrical shell and the ECM antennas as line sources.

This model is simply a dielectric, cylindrical shell of circular cross section with an external line source. The model does not account for the metallic fuselage, but it may still lend some insight to how electromagnetic radiation scatters from large, curved structures. In addition, this model and approach may bring out some of the problems which will be encountered in trying to answer the question of lobing due to large structures.

III. The Basic Theory

A. Separation of Variables/Eigenvalue Method

The geometry to be analyzed is shown in Figure 6. It is assumed that the cylindrical shell and the line source are infinite in z and that there is no z-variation in any quantity. All quantities have an $e^{j\omega t}$ time dependence assumed and suppressed. Since the source is a z-directed electric current, the potential due to the source is a vector potential $\overline{A}$, with

$$A_z \neq 0 \quad (\overline{F} = 0;\ A_\rho,\ A_\phi = 0) \tag{1}$$

Note that in this study $\overline{H} = \overline{\nabla} \times \overline{A}$ which is consistent with Harrington (Ref. 2). The current on the wire is given by (also see Appendix B)

$$\overline{J} = I\ \delta(\overline{r})\ \hat{z} \tag{2}$$

It is assumed that the permeability in each region is that of free space. Also, it is assumed that the dielectric material is a perfect insulator, having a zero loss tangent.

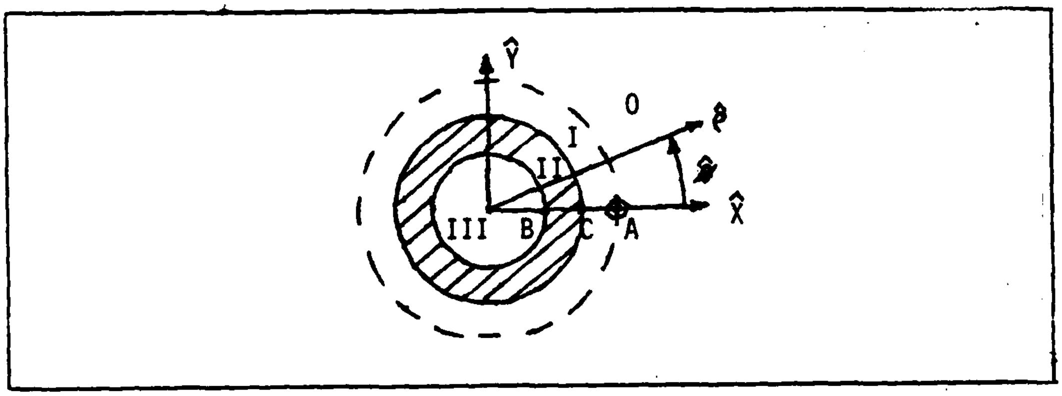

Figure 6: Model for the eigenvalue solution, including the numbered regions.

The fact that there is no variation with z and that there is only a nonzero scalar potential given by (1) implies that A_z must be a solution to the

scalar Helmholts equation for source-free regions (3), where the surfaces in Figure 6 are not in any region but present boundary conditions to those regions.

$$\overline{\nabla}^2 A_z + k^2 A_z = 0 \tag{3}$$

A solution to (3) in cylindrical coordinates is (Ref. 2:199)

$$A_z = B_n(k_\rho \rho)\, h_n(\phi) \tag{4}$$

where

$$\frac{\partial}{\partial z} = 0,\ k_z = 0 \tag{5}$$

$B_n(k_\rho \rho)$ is some solution to Bessel's equation of order n and $h_n(\phi)$ is sin ϕ, cos ϕ, or some linear combination thereof. The separation parameter equations are

$$k_{\rho_i}^{\ 2} + k_z^{\ 2} = k_i^{\ 2};\ i = 0,\ \mathrm{I},\ \mathrm{II},\ \mathrm{III} \tag{6}$$

substituting (5) into (6) yields

$$k_{\rho_i} = k_i;\ i = 0,\ \mathrm{I},\ \mathrm{II},\ \mathrm{III} \tag{7}$$

From (1) and (5) the electric field intensity can be written as

$$E_z = -\hat{Z}_0\, A_z \tag{8}$$

where

$\hat{Z}_0 = j\omega\hat{\mu}$. The magnetic field is

$$\overline{H} = \frac{\partial A_z}{\partial \rho}\,\hat{\phi}\ \frac{1}{\partial}\ \frac{\partial A_z}{\partial \phi}\ \hat{\rho} \tag{9}$$

For the continuous geometry shown in Figure 6, one would (correctly) expect the potential to have even symmetry about the origin, hence a cosine variation in ϕ.

$$A_{z_i} = B_n(k_i \rho)\cos(n\phi),\ n \text{ an integer} \tag{10}$$

From (8) and (10) the total field for a given region, i, and azimuthal index,

n, is given by

$$E_{z_{i,n}} = j\omega\mu_0 \, B_n(k_i\rho) \cos(n\phi) \tag{11}$$

Referring to Table 5.1 (Ref. 2:203), the potentials for the respective regions can be written as follows:

REGION	ELECTRIC VECTOR POTENTIAL	
0	$A_{z_{0,n}} = F_n H_n^{(2)}(k_0\rho)\cos(n\phi)$	(a)
I	$A_{z_{I,n}} = [A_n J_n(k_0\rho) + B_n N_n(k_0\rho)]\cos(n\phi)$	(b)
II	$A_{z_{II,n}} = [C_n J_n(k_2\rho) + D_n N_n(k_2\rho)]\cos(n\phi)$	(c)
III	$A_{z_{III,n}} = E_n J_n(k_0\rho)$	(d)

(12)

where

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0} = \frac{2\pi f}{C} = \frac{2\pi}{\lambda_0} \tag{13}$$

$$k_2 = \omega\sqrt{\mu_0\varepsilon_0} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = k_0\sqrt{\epsilon_r} \tag{14}$$

and

$H_n^{(2)}$ the Hankel function of order n of the second kind

J_n is the Bessel function of the first kind

N_n is the Neuman function (also called the Bessel function of the second kind)

These potentials automatically satisfy appropriate boundary conditions at the origin and the radiation condition. Equation (16) contains six unknown constants for a given order n, $A_n \ldots F_n$. To solve for these six unknowns the following general boundary conditions are applied (see Figure 7):

$$\hat{n} \times (\overline{H}_i - \overline{H}_{i+1}) = \overline{J}_s$$

$$(\overline{E}_i - \overline{E}_{i+1}) \times \hat{n} = \overline{M}_s \tag{15}$$

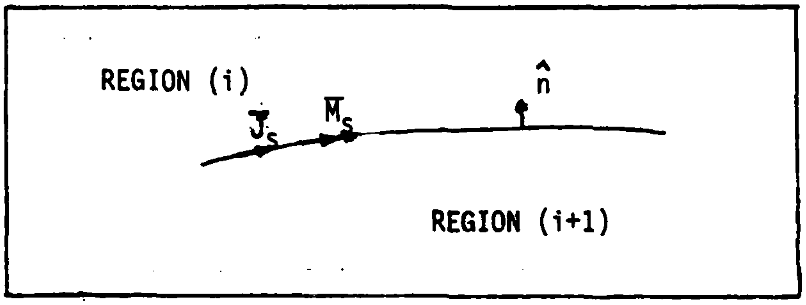

Figure 7. Geometry for the boundary conditions over a surface S.

Since the obstacle (the dielectric, cylindrical shell) is not a magnetic conductor and is assumed to be perfectly insulating, the magnetic and conduction current are neglected. With this the following six boundary conditions result:

$$\hat{n} \times \overline{E}_{i,n} = \hat{n} \times \overline{E}_{(i+1,n)}, \quad i = 0,\ I,\ II \quad @\ \rho = a,\ c,\ b \text{ respectively} \quad (a)$$

$$\hat{n} \times \overline{H}_{i,n} = \hat{n} \times \overline{H}_{(i+1,n)}, \quad i = I,\ II \quad @\ \rho = c,\ b \text{ respectively} \quad (b) \quad (16)$$

$$n \times \overline{H}_{o,n} - n \times \overline{H}_{I,n} = \overline{J}_s, \quad @\ \rho = a \quad (c)$$

where $\overline{J}_s = \frac{I}{a}\delta(\phi)\,\hat{z}$ on the $\rho = a$ surface as shown in Appendix B. The goal is to determine the far-zone radiation pattern from the line source in the presence of the obstacle. This field will be given by a sum of the field given by (11) and (12a) over all possible modes.

$$E_z(\rho_o, \phi_o) = j\omega\mu \sum_{n=o}^{\infty} F_n H_n^{(2)}(k_o\rho_o) \cos(n\phi.) \qquad (17)$$

The observation point for the field in the far zone is (ρ_o, ϕ_o). From (17) it is evident that the only coefficient that need be solved for is F_n. This means that once the problem is solved for F_n for each n from zero to infinity, the radiation at all points in region 0 can be directly obtained.

By applying (16) six equations in six unknowns result (see Appendix B) which can be arranged into the follow matrix equation:

$$
\begin{bmatrix}
J_n(k_o a) & N_n(k_o a) & 0 & 0 & 0 & -H_n^{(2)}(k_o a) \\
J_n(k_o c) & N_n(k_o c) & -J_n(k_2 c) & -N_n(k_2 c) & 0 & 0 \\
0 & 0 & J_n(k_2 b) & N_n(k_2 b) & -J_n(k_o b) & 0 \\
k_o J_n'(k_o c) & k_o N_n'(k_o c) & -k_2 J_n'(k_2 c) & -k_2 N_n'(k_2 c) & 0 & 0 \\
0 & 0 & k_2 J_n'(k_2 b) & k_2 N_n'(k_2 b) & -k_o J_n'(k_o b) & 0 \\
J_n'(k_o a) & N_n'(k_o a) & 0 & 0 & 0 & -H_n^{(2)'}(k_o a)
\end{bmatrix}
x
\begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \\ E_n \\ F_n \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-I}{2\pi k_o a} \end{bmatrix}
\quad (18)
$$

Matrix equation (18) can be solved to obtain coefficients $A_n \ldots F_n$ for a given order n.

Richmond (Ref. 23:338) gives results for scattering by the cylindrical shell in terms of echo width per wavelength. To make a comparison to his results the equation for the normalized echo width for this eigenvalue solution is derived. In a two-dimensional problem having linear polarization as that considered here, the echo width can be defined by (Ref 2:358)

$$
L_e = \lim_{\rho \to \infty} \left(2\pi\rho \left| \frac{\overline{E}^s}{\overline{E}_r^i} \right|^2 \right) \quad (19)
$$

The total field at some point in the far zone is the sum of the incident field at that point due to sources without the obstacle present and the scattered field due to the polarization currents impressed upon the obstacle by the sources

$$
\overline{E} = \overline{E}^s + \overline{E}^i \quad (20)
$$

The scattered field in (19) is determined by (20). The reference field, $\overline{E}_r^i$, in (19) will be assumed to be the incident field at the center of the obstacle without the obstacle present, for consistency with Richmond (Ref. 23).

The large argument asymptotic expansion for the Hankel function of the

second kind (Ref. 25:364)

$$H_n^{(2)}(k_o\rho_o) \xrightarrow[\substack{k_o\rho\to\infty \\ |k_o\rho|>>n}]{} \sqrt{\frac{2}{\pi k_o\rho_o}}\; e^{-j[k_o\rho_o - (2n+1)\pi/4]} \tag{21}$$

can be used to rewrite the total field as

$$E_z(\rho_o,\phi_o) \approx -j\omega\mu \sqrt{\frac{2}{\pi k_o\rho_o}}\; e^{-jk_o\rho_o} \sqrt{j} \sum_{n=o}^{\infty} F_n e^{j\frac{n\pi}{2}} \cos(n\phi_o) \tag{22}$$

Recall that in (22) the far-zone observation point (ρ_o,ϕ_o) is relative to the origin at the center of the obstacle, and

$$\rho_o = \frac{2D^2}{\lambda} \tag{23}$$

where D is the diameter of the obstacle.

The incident field, $\overline{E}^i$, can be written as (Ref. 2:236)

$$E_z^i(\rho,\phi) = -\frac{k_o^2 I}{4\omega\epsilon_o}\; H_o^{(2)}(k_o\rho_n) \tag{24}$$

In (24) ρ_n is the distance from the source to the observation point (ρ,ϕ), or equivalentally (X,Y), and is written as

$$\rho_n = \sqrt{(X-X')^2 + (Y-Y')^2} \tag{25}$$

which in the far-zone can be written as (see Appendix C)

$$\rho_n \approx \rho - a\cos\phi \tag{26}$$

Again, the large argument asymptotic expansion can be used

$$H_o^{(2)}(k_o\rho_n) \xrightarrow[k_o\rho_n\to\infty]{} \sqrt{\frac{2}{\pi k_o(\rho - a\cos\phi)}}\; e^{-j[k_o(\rho - a\cos\phi)-\pi/4]} \tag{27}$$

In (32) $\rho>>a$ which allows (32) to be written as

$$H_o^{(2)}(k_o\rho_n) \xrightarrow[k_o\rho_n\to\infty]{} \sqrt{\frac{2}{\pi k_o\rho}}\; e^{-jk_o\rho} \sqrt{j}\; e^{jk_o a\cos\phi} \tag{28}$$

Substituting (28) in (29)

$$E_z^i(\rho,\phi) \approx \frac{-k_o^2 I}{4\omega\varepsilon_o} \sqrt{\frac{2}{\pi k_o \rho}}\; e^{-jk_o\rho} \sqrt{j}\; e^{jk_o a\cos\phi} \tag{29}$$

From (17), (20), and (29) the scattered field at the far-zone point is

$$E_z^s(\rho,\phi) = \sqrt{\frac{2}{\pi k_o \rho}}\; e^{-jk_o\rho} \sqrt{j}\; (-j\omega\mu) \sum_{n=o}^{\infty} F_n\, e^{\frac{jn\pi}{2}} \cos(n\phi) \tag{30}$$

The incident field at the center of the obstacle without the obstacle present, $\overline{E}_r^i$, is also given by (24), but ρ_n now becomes the distance from the source to the origin

$$\rho_n = |a|$$

The large argument asympototic expansion cannot be used in this case since the observation point is not in the far-zone. $\overline{E}_r^i$ can be written as

$$E_{z_r}^i = \frac{-k_o^2 I}{4\omega\varepsilon_o} \left[J_o(k_o a) - j\, N_o(k_o a)\right] \tag{31}$$

where

$$H_n^{(2)}(z) = J_n(z) - j\, N_n(z) \tag{32}$$

Substituting (30) and (31) into (19), dividing by λ_o, and simplifying, the echo width normalized to the free space wavelength can be written as

$$Le/_{\lambda_o} = \frac{8\varepsilon_o^2 c^4}{\pi^3 f^2 I^2} \; \frac{1}{J_o^2(k_o a) + N_o^2(k_o a)} \left| -j\omega\mu \sum_{n=o}^{\infty} F_n\, e^{j\frac{n\pi}{2}} \cos(n\phi) + \frac{\omega\mu_o I}{4}\; e^{j(k_o a\cos\phi)} \right|^2 \tag{33}$$

where

$$C = {}^{1}/_{\mu_o\varepsilon_o} \approx 3 \times 10^8 \text{ m/sec} \tag{34}$$

Equation (33) is a relatively simple representation for the echo width per wavelength. To evaluate the equation for specific parameters in principle requires evaluation of an infinite sum involving constants, F_n, which

must each first be calculated by solution of (18). The analysis to this point presumes the existance of an "exact solution" represented as an infinite summation. If this infinite summation is an exact representation of a sable situation, it must be covergent to the exact solution. If the summation is absolutely convergent, truncation after a finite number of terms will result in a bounded error. The magnitude of the error will depend on the number of terms retained. This method of evaluation through truncation is made use of in the program in Appendix F. The error produced by truncation will be addressed later in this paper.

B. Moment Method Solution

In the previous section a solution for the scattering by the dielectric obstacle was obtained from an exact solution to the Helmholtz equation. Consider now an inhomogeneous region V containing source $\overline{J}^i$ and $\overline{M}^i$ as shown in Figure 8.

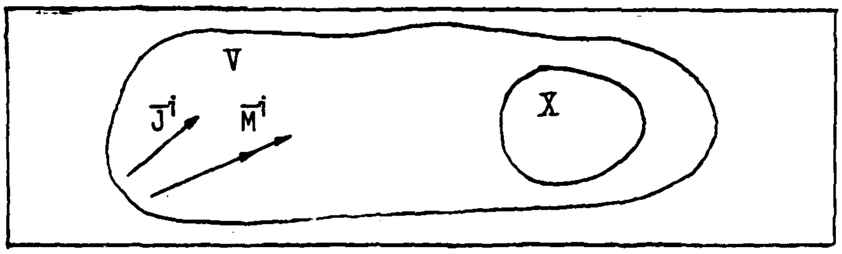

Figure 8. Region V containing sources and obstacles

Within region V the following must hold

$$\begin{aligned} -\overline{\nabla} \times \overline{E} &= \hat{z}_o \overline{H} + \overline{M}^i \\ \overline{\nabla} \times \overline{H} &= \hat{y}_o \overline{E} + \overline{J}^i \end{aligned} \tag{35}$$

Where the admittivity, $\hat{y}_o = j\omega\varepsilon_o$, and the impedivity, $\hat{z}_o = j\omega\mu_o$, are functions of position. If most of the region V is homogeneous except for some small subregion(s), X, region W can be defined as all of V excluding X. Within region W then $\hat{z}_{o_W}$ and $\hat{y}_{o_W}$ are constant. Equation (35) can be written as

$$\begin{aligned} -\overline{\nabla} \times \overline{E} &= \hat{z}_{o_W} \overline{H} + \overline{M}^e \\ \overline{\nabla} \times \overline{H} &= \hat{y}_{o_W} \overline{E} + \overline{J}^e \end{aligned} \tag{36}$$

where

$$\begin{aligned} \overline{M}^e &= (\hat{z}_o - \hat{z}_{o_W}) \overline{H} + \overline{M}^i \\ \overline{J}^e &= (\hat{y}_o - \hat{y}_{o_W}) \overline{E} + \overline{J}^i \end{aligned} \tag{37}$$

$\overline{J}^e$ and $\overline{M}^e$ are the effective currents and can be treated as source currents radiating in a homogeneous region. For the problem being considered $\hat{z}_{o_W}$ and

$\hat{y}_{o_w}$ are the free space parameters with $\hat{\mu} = \hat{\mu}$ and $\sigma \sim 0$ so that

$$\overline{M}^e = \overline{M}^i$$
$$\overline{J}^e = j\omega(\hat{\varepsilon}-\varepsilon_o)\ \overline{E} + \overline{J}^i \tag{38}$$

The situation shown in Figure 9 is equivalent then to that of Figure 8.

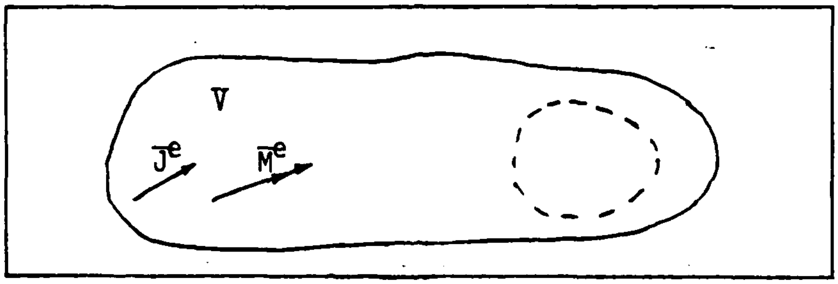

Figure 9. Equivalent sources radiating in a homogeneous region V.

The total field in region V is given by

$$\overline{E} = \overline{E}^i + \overline{E}^s \tag{39}$$

The incident field, $\overline{E}^i$, upon the obstacle (subregion X) is produced by $\overline{J}^i$ outside the obstacle while the scattered field, $\overline{E}^s$, is produced by polarization currents, $\overline{J}^p$, induced throughout the obstacle.

$$\overline{J}^p = j\omega(\varepsilon-\varepsilon_o)\ \overline{E} \tag{40}$$

In (40) $\overline{E}$ is the field induced throughout the obstacle.

As in the development of the eigenvalue solution, the incident field, from the electric current filament parallel to the z-axis, will have only an $\hat{z}$-component. The $\hat{z}$-component of incident field will produce a scattered field having only an $\hat{z}$-component and meeting all boundary conditions. From (39) the total field will also have only an $\hat{z}$-component. The result then is an incident field from the filament source (see Figure 6) in all space. The obstacle can be replaced by the polarization currents induced on the obstacle radiating in free space. The superposition of the fields produced

is the total field existing in free space.

The field from an electric current filament is given by

$$d\overline{E} = -\frac{k^2}{4\omega\varepsilon_o} \; H_o^{(2)} \; (k\rho) \; dI \; \hat{z} \tag{41}$$

From (40) the scattered field is

$$dE_z^{\,s} = -\frac{k^2}{4\omega\varepsilon_o} \; H_o^{(2)} \; (k\rho) \; dI \tag{42}$$

where

$$dI = \overline{J}^p \, ds' = j\omega(\hat{\varepsilon}-\varepsilon_o) \; \overline{E} \; ds' \tag{43}$$

and

$$k^2 = k_o^{\,2}$$

In (43) $\overline{E}$ is the total field within the obstacle which has only a $\hat{z}$-component, and ds' is the increment of surface area on the cross section of the obstacle. (The prime on ds' indicates source coordinates.) Integrating (41) over the surface of the obstacle and substituting (43) for dI, the scattered field from a dielectric of low loss tangent is

$$\begin{aligned} E_z^{\,s}(x,y) &= -\frac{k_o^{\,2}}{4\omega\varepsilon_o} \iint_{s'} j\omega(\varepsilon-\varepsilon_o) \; E_z(x',y') \; H_o^{(2)} (k\rho) \; dx'dy' \\ &= -\frac{jk^2}{4} \iint_{s'} (\varepsilon_r-1) \; E_z(x',y') \; H_o^{(2)} \; (k\rho) \; dx'dy' \end{aligned} \tag{44}$$

where

$$\varepsilon_r = {}^{\varepsilon}/_{\varepsilon_o} \tag{45}$$

$$\rho = \sqrt{(x-x')^2 + (y-y')^2} \tag{46}$$

and (x,y) is the observation point while (x',y') is the source point.

From (39) and (44)

$$E_z(x,y) = E_z^{\,i}(x,y) + \left(-\frac{jk^2}{4}\right) \iint_{s'} (\varepsilon_r-1) \; E_z(x',y') \; H_o^{(2)} \; (k_o\rho) \; dx'dy' \tag{47}$$

The procedure now is to divide the cross section of the obstacel into N

'square' cells small enough so that ε_r and the field intensity are essentially constant over each cell (refer to Figure 10). Enforcing (44) at the center of each cell m results in

$$E_{z_m} + \frac{jk^2}{4} \sum_{n=1}^{n} (\varepsilon_n - 1) E_{z_n} \iint_{\text{cell } n} H_o^{(2)} (k_o \rho) \, dx'dy' = E_{z_m}^{i} \tag{48}$$

where

$$\rho = \sqrt{(x'-x_m)^2 + (y'-y_m)^2} \tag{49}$$

$$\varepsilon_n = \varepsilon_r (x_n, y_n) \tag{50}$$

$$E_{z_n} = E_z(x_n, y_n) \tag{51}$$

and E_{z_n} is the field intensity at the center of each cell n.

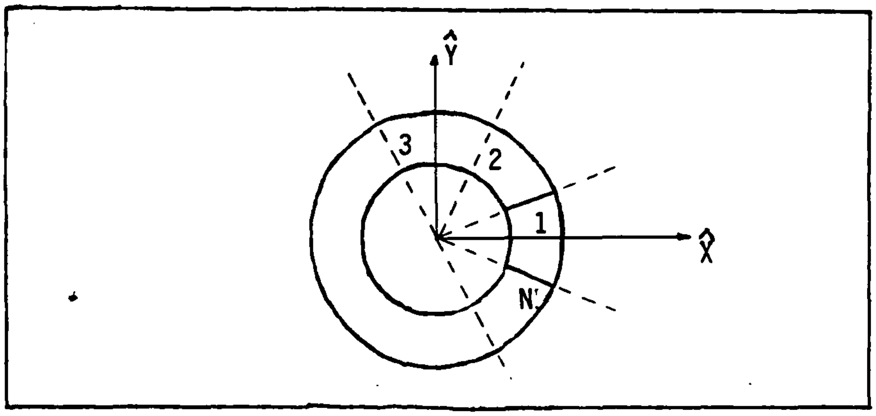

Figure 10. Dividing the obstacle into N nearly square cells.

Equation (48) states that the total field at some cell m is due to the incident field at that cell plus the scattered field from all N cells (including cell m) at cell m. Yet, (48) is one equation with N unknowns. To solve for these N unknown (48) is enforced at the center of all N cells, resulting in N equations in N unknowns. This process of enforcing the integral equation (44) at the centers of N cells is called Point-Matching and is a special case of the method of moments (Ref. 21:312).

The evaluation of (48) involves an integration over a square surface of

the Hankel function of the second kind of order zero. A singularity exists when the argument of the Hankel function approaches zero, which occurs when the source point approaches the observation point. This singularity is integrable (see Appendix D) if the square surface is replaced by a circular surface of equal area with radius a_n. The result is

$$\frac{jk^2}{4} \underset{\text{cell } n}{\int_0^{2\pi} \int_0^{a_n}} H_0^{(2)} (k_o\rho)\, \rho'\, d\rho'\, d\phi'$$

$$= (j/2)\, [\pi k_o a_n H_1^{(2)}(k_o a_n) - 2j],\ m = n \qquad (52)$$

$$= \frac{j\pi k_o a_n}{2}\, J_1(k_o a_n)\, H_o^{(2)}(k_o \rho_{mn}),\ m = n$$

where ρ is given by (49) and

$$\rho_{mn} = \sqrt{(X_m - X_n)^2 + (Y_m - Y_n)^2} \qquad (53)$$

The polar coordinates ρ' and ϕ' are based on a coordinate origin at the center of cell n, the source point contributing to the total scattered field at cell m.

For the geometry shown in Figure 6 the matrix equation given by (47) will be symmetric applying (52) for m = 1,2,3,..,N. It is obvious that the diagonal terms, m = n, will be equal and the off-diagonal terms depend only upon the distance between the source and observation point. The symmetry inherent in this geometry results in a Toeplitz matrix (Ref. 21:340). This specialization to a Toeplitz matrix is very advantageous when inversion of large matrices is necessary and will be discussed later in this paper.

Once the matrix equation has been solved for the field induced within the dielectric (44) can be used to obtain the scattered field in the far field. Applying the method of point matching, (44) can be rewritten as

$$E_z^{s}(x,y) = -\frac{j\pi k_o}{2}\sum_{n=1}^{N} (\epsilon_n - 1)\, E_n a_n J_1\,(k_o a_n)\; H_o^{(2)}(k_o \rho_n) \tag{54}$$

Note that in (54) the fact that the observation point is in the far field guarantees that it will be different from the source point on the obstacle so that the appropriate integral for $m \neq n$ of (52) is used and

$$\rho_n = \sqrt{(X - X_n)^2 + (Y - Y_n)^2} \tag{55}$$

From Appendix C the distance from the source point on the obstacle, (X_n, Y_n), to the observation point in the far field, (X,Y), can be written as

$$\rho_n = \rho_o - X_n \cos\phi - Y_n \sin\phi \tag{56}$$

where ρ_o and ϕ are the polar coordinates of the observation point in the far field. Because the observation point is in the far field, the large argument asymptotic expansion for the Hankel function of order zero (Ref. 25:364) given by (57) can be used.

$$H_o^{(2)}(k\rho) \;\rightarrow \sqrt{\frac{2}{\pi k \rho}}\; e^{-j[k\rho - \pi/4]} \tag{57}$$

Since $\rho_o \gg X_n$ and $\rho_o \gg Y_n$, the result is

$$H_o^{(2)}(k_o \rho_o) \rightarrow \sqrt{\frac{2}{\pi k_o \rho_o}}\; e^{-j[k_o(\rho_o - X_n\cos\phi - Y_n \sin\phi) - \pi/4]} \tag{58}$$

Substituting (58) into (54).

$$\begin{aligned} E_z^{s}(X,Y) \approx -j\sqrt{\frac{\pi k_o}{2\rho_o}}\; e^{-jk_o\rho_o} \sum_{n=1}^{N} (\epsilon_n - 1) E_n a_n J_1(k_o a_n) \\ \cdot e^{j[k_o(X_n \cos\phi + Y_n \sin\phi) + \pi/4]} \end{aligned} \tag{59}$$

As was done for the eigenvalue solution, the echo width per wavelength can be determined using (59) and (19), which is rewritten here for convenience

$$L_e/\lambda = 1/\lambda \lim_{\rho_o \to \infty} \left(2\pi\rho_o \left|\frac{\bar{E}^{s}}{\bar{E}^{i}}\right|^2\right) \tag{19}$$

The incident field referred to in (19) is a constant field intensity to which the scattered field is normalized. So that comparison of results can be made to Richmond (Ref. 23:338), this incident field is taken to be that at the center of the dielectric shell with the obstacle removed, radiated from the electric filament line source. This incident electric field is given by Harrington (Ref. 2:236) and is

$$E_z^{\,i} = -\frac{k_o^{\,2} I}{4\omega\varepsilon_o} H_o^{(2)}(k_o|\rho-\rho'|) \tag{60}$$

where $|\rho-\rho'|$ is the absolute distance from the source to the center of the obstacle and is relatively small compared to ρ_o. Therefore, (19) is written as

$$L_e/_\lambda = \lim_{\rho_o\to\infty} \left(2\pi\rho_o \left| \frac{-j\,\dfrac{j\pi k_o}{2\rho_o}\, e^{-jk_o\rho_o} \sum_{n=1}^{N} (\varepsilon_n-1)E_n a_n J_1(k_a a_n) e^{j[k_o(X_n\cos\phi+Y_n\sin\phi)]}}{\dfrac{-k_o^{\,2} I}{4\omega\varepsilon_o} H_o^{(2)}(k_o|\rho-\rho'|)} \right|^2\right)$$

$$= \pi^2 k_o \left| \frac{\sum_{n=1}^{N} (\varepsilon_n-1)E_n a_n J_1(k_o a_n) e^{j[k_o(X_n\cos\phi+Y_n\sin\phi)]}}{\dfrac{-k_o^{\,2} I}{4\omega\varepsilon_o} H_o^{(2)}(k_o|\rho-\rho'|)} \right| \tag{61}$$

It is also possible to obtain the total field in the far field by adding (60), evaluated at $\rho = \rho_o\hat{\rho} + \phi\hat{\rho}$, and (58). It should be noted that the far field is determined using

$$\rho_o = \frac{2D^2}{\lambda} \tag{62}$$

where D is the diameter of the obstacle and λ is the free space wavelength.

IV. Procedure to Arrive at Numerical Results

A. The Eigenvalue Solution

The theory and derivation behind both the eigenvalue solution and the moment method solution are not overly complex, but several complexities were realized in attempting to obtain numerical results from these solutions. The eigenvalue solution involves an infinite series of coefficients multiplying Hankel functions. The coefficients are arrived at by solving six simultaneous equations. Each equation is comprised of Bessel functions of the first and second kind, Hankel functions of the second kind, and the derivatives of each. How to calculate the Bessel and Hankel functions, how to perform the matrix inversion and how to work with the infinite series were the three major questions to be answered to obtain numerical results from the eigenvalue solution.

Calculating the Bessel and Hankel Functions

Evaluation of the eigenvalue solution requires calculation of Bessel functions of integer order and of widely varying argument. An argument as small as 1.57, involved in obtaining results to compare to Richmond's results (Ref. 23:338), had to be handled as well as those as large as 190, to produce answers for a scatterer of 60 wavelengths cross section. Another concern, in addition to range, was how to calculate the value of these functions efficiently. The matrix equation (23) would theoretically have to be solved an infinite number of times which meant evaluating the elements of the matrix an infinite number of times. Easily, a routine to calculate these functions could consume too much CPU time to approximate the summation. A third area of concern was for the accuracy in evaluating the functions.

A routine based on equations given by Abramowitz (Ref. 25:369) was first obtained. This routine involved calculating Bessel functions of the first and second kind of order zero and one using polynomial approximations appropriate to the argument. Using recurrence relations, values for the functions at other orders were calculated. The values from this routine were the same to an average of 6 decimal places as those of Table 9.4 (Ref. 25:407) for arguments less than 3. When the argument was increased to 100 accuracy was maintained at an average of 6 decimal places except at the point where the order also became 100. From an order of 100 to about 104 the accuracy dropped to an average of 4 decimal places for an argument of 99.95 (comparisons made with those given by Aiken (Ref. 26)). Then the accuracy returned to an average of 6 decimal places for order greater than 104. The program was checked to determine why such a lack of accuracy existed for the case of large argument and order being equal. The program followed exactly from the equations given by Abramowitz and no programming errors were detected. One change which was made was to increase the point at which recurison downward began for evaluation of the Bessel function of the first kind. The program had been designed to begin recurison from an order 10 greater than the order desired. That starting point was increased to 20, 40, and 80 above the order desired. No change at all was detected by increasing even to the 40 point, but the change to the 80 point resulted in the development of significant errors. The errors were seen as exponents of -23 multiplying the value of the function for argument and order being large and equal. This error could have been the result of the normalization value (Ref. 25:385) becoming very large and exceeding the limit of the machine when divided into the trial values the result was very small numbers of exponent -23.

The problem with the previous Bessel routine was not investigated

further mainly because it was not an economical routine to use. It would return values of $J_n(x)$ and $N_n(x)$ for only one order at a time and would therefore have to be called some 2100 times for the scatterer of sixty wavelengths. Another routine was obtained; it calculated the zero and first order terms in the same way as the previous routine did, but it used what is called "continued fraction formulae" (Ref. 27:153). This approach results directly from the recurrence relations, and it produced no more accuracy than the preceeding routine except at equal large argument and larger order. At that point accuracy was maintained at an average of 4 to 5 decimal places. The attractive feature of this routine was that it would pass an array of values starting from the order desired down to order zero, resulting in fewer required calls. Yet, because of the array size and the number of arrays required, core limitations were exceeded on the machine used. Another problem with this routine was that it would not return correct values for the Bessel function of the second kind having arguments less than 17.5. This problem was corrected, but because the routine actually consisted of four separate subroutines and a lot of core was required, the first routine was used to try to obtain some type of answers for this eigenvalue solution.

Matrix Inversion

With the matrix formed it now had to be inverted to solve for the coefficients. There are several methods available in the literature to accomplish matrix inversion; Gaussian Elimination, Gaus-Seidel, and Gaus-Jordan are three typical methods (Ref. 24, 25). But, for matrices of order n>2 a method known as Cholesky's method (also named Crout reduction) "requires fewer arithmetic operations" than either of the three mentioned methods, "making it the fastest of the basic elimination methods" (Ref. 24:198). It is a lower-upper triangular decomposition method and can be made economical of

core by storing the lower and upper triangular matrices in the same area allocated to the matrix to be inverted. This method results in a simple, fast program for matrix inversion.

The Infinite Series

As discussed in the development of the eigenvalue solution, the infinite summation must converge to the exact answer. In order to determine the accuracy of the truncated summation, the behavior of the coefficient for increasing order must be considered. A definite bound on the error due to truncation was not determined, yet the computer generated numbers for the coefficient F_n did show that F_n became purely imaginary and then approached zero for increasing order, as was predicted. Approximately 350 terms were needed in the sum before additional terms became insignificant relative to some ε for a structure of 60 wavelength diameter. It was opted to proceed with the programming, working with the truncated infinite series as described, in order to obtain some type of results.

B. The Moment Method Solution

The most serious obstacle in obtaining numerical results from the moment method solution was the demand for core. The matrix to be inverted became quite large for a large geometry since the number of cells must increase to maintain accuracy in the final answer. Also, to keep the program as general as possible so that characteristics of the obstacle, such as size and shape, might be changed, the program was written as a collection of separate subroutines. This programming method called for other arrays in addition to the matrix to be set up which quickly exceeded the amount of core available to the program. Fitting the program in the available core and obtaining

reasonable execution time were two pressing problems in programming the moment method solution.

Core Requirements

In developing the theory for the moment method solution, it was mentioned that the symmetry of this particular problem would yield a Toeplitz matrix. Numbers were generated for a simple case and showed that indeed the matrix was Toeplitz in form. This was a significant observation since routines exist which take advantage of this special form. The routine chosen to be used (see Appendix G) does not require the passing of the entire matrix to be inverted but only the first row or first column of the matrix. Overall the routine requires only five arrays of dimension n where n is the dimension of the square matrix. This results in a significant reduction in the demand for core.

To combat the core requirements due to programming by use of subroutines a feature called Extended Memory Area (EMA) was utilized. All arrays were first defined to be in a common block; that common block was then defined to be in EMA. Basically, EMA is disk memory as opposed to core memory. A compiled program is loaded into core to be run, and arrays which are not specified to be in EMA are allocated space in core. Only so much core can be taken up by any particular program, 64K for the HP 21MX M series; if more core is asked for, the loader aborts and the program cannot be run. EMA allows the program to be loaded and run, but it also has limitations.

Processing Time

EMA, in conjunction with designating a large background when loading, will allow a large program to be loaded and run, but it also requires more

processing time. The system has to swap data in EMA from disk to core in order to process the data as directed by the main program. This swapping takes up much more computer time than working with data already in core. For the size of the arrays required to analyze a large structure of 60 wavelength cross section run time exceeded two hours. Obviously, this was a costly program to run on this minicomputer as programmed.

C. The Priority Set for Both Methods

The intent was still to produce two independent computer solutions for the scattering from a dielectric cylinder of circular cross section and to form an argument for their validity. In an attempt to meet this goal the preceding concerns were incorporated into two programs. Not all questions were answered nor were all the concerns met. A discussion of the results from these two programs follows along with conclusions and recommendations on how to improve upon them.

V. Discussion of Results

Computer generated output appears in Appendix E, including plots of the scattered and total field for a number of trials. As emphasized throughout this paper, the goal has been to generate numerical results from the two methods discussed, to compare those results to Richmond's results (Ref. 23: 338), and to increase the size of the structure to model an actual radome. In the process an argument for the validity or invalidity of the answers plus a discussion of problems encountered would be presented. Looking at Figures E-2, E-3, and E-4 along with Table E-1, it is evident that the results are very close to being the same for the small structure of 0.6 wavelength cross section. The fact that equivalent results were obtained from the two completely different methods seems to indicate that the error due to the truncation of the sum in the eigenvalue solution was not a significant determinant of the final answer. Both programs calculated the Bessel functions using the same routine with the accuracy problems discussed previously. The results thus obtain credibility because these two methods, which were independent and very different, resulted in comparable answers.

Before ever attempting any calculations one would probably assume that a small obstacle of 0.6 wavelength cross section would cause very little disturbance of the field incident upon it. Figures E-5 and E-6 are consistent with this assumption, showing a very smooth, slight variation over 360°. This structure does not possess any abrupt changes in curvature; therefore, a wave which might be trapped by the structure would radiate consistently as it traveled around the structure. As the trapped wave radiated it would be losing energy and would in turn have less and less to radiate. What difference a trapped wave would present is not evident in this trial, but this

argument is consistent with the smooth variation of the total field. It is also possible to view this problem as a superposition of a circular array of line dipoles and a line source radiating in free space. This is a physical picture of the moment method solution. Two dipoles having some current distribution on them and spaced a distance d apart will produce nulls in their radiation pattern about $^{\lambda}/d$ radians apart. The maximum distance d for this first trial is $2c = 0.6\lambda$. Therefore, nulls should appear in the scattered field spaced by approximately 1.67 radians or 95.5°. Looking at Figure E-3, the nulls appear to be spaced by about 110°.

The outer diameter of the obstacle was increased 10 fold for the second trial. The thickness of the dielectric was kept at 0.05λ and the distance from the outside of the obstacle to the location of the line source was kept at 0.2λ. Since the first trial compared closely using 37 cells to approximate the cylindrical shell, 370 cells was used in the second trial to maintain close to the same cell density per wavelength. A comparison of Figures E-7 and E-8 and Figures E-9 and E-10 shows equivalent results existing again for the two different methods. A couple of slight differences do exist though. First, the magnitude of the results from the moment method solution are noticably less than those from the eigenvalue solution. Round off errors, incorrect evaluation of the Bessel functions, and/or a nonoptimum number of cells could account for this small difference. The second difference is that the total field of Figure E-9 shows a sudden drop at about 152.5° and then a sudden increase about 2.5° later. This also occurs at about 207.5°. Figure E-10 shows a smooth variation in the total field in these regions. The eigenvalue results do not show a squiggle in the pattern at 172.5° and 187.5°, as do the moment method results. Both polar plots were plotted at every 1° so the difference is not due to resolution.

An interesting observation can be made by overlaying Figures E-5 and E-9 and Figures E-6 and E-10 as shown in Figures E-11 and E-12 respectively. After increasing the cylinder diameter the result appears to be that the general shape of the total field pattern remains the same. Effectively, the circular array spoken of previously has been increased in size and more dipoles added. The nulls should now be spaced about $^{\lambda}/6.0\lambda = 0.167$ radians or 9.55° apart. The addition of dipoles to the array would cause the variation in the gain pattern as seen. With the observations pointed out, it is felt that the results of this trial are also correct.

The third trial was for another 10 fold increase in the geometry. The results from the eigenvalue solution appear in Figures E-13 and E-14. It was not possible to run the moment method program because the neeo for 3700 cells over extended EMA. An attempt was made to break the obstacle into 3700 cells and to work with on every other cell in computing the total field. The program with this modification would then load and run, but after three hours of run time the program was aborted. (This moment method program slowed the system down considerably.) Because there is nothing to compare the results of the eigenvalue solution against, only general comments on those results can be made.

Applying the array analysis, the nulls should appear at $^{\lambda}/60\lambda = 0.0167$ radians or 0.955°. Observing Figure E-12 there are indeed 20 peaks in 20°. A change in the scattered field as compared to the previous two trials is the maximum occuring at zero degrees. This maximum is plotted at about 0.95 in Figure E-13. Having normalized all calculations of the scattered field relative to the maximum magnitude of scattered field, the maximum as plotted should be 1.0 as for trials one and two. This evident error immediately sheds doubt on the validity of the results. Another concern is that the

scattered field pattern has such deep nulls and so many of them. It is hard to predict what is causing such a spiked pattern to exist. Symmetry still holds for the total field in Figure E-14, but again it is difficult to validate the pattern with nothing else to compare to.

VI. Conclusions and Recommendations

The fact that the two different programs produced results comparable to published results for the small structure of 0.6 wavelength diameter gives confidence that these results are correct, thus validating both the approach and the computer program. When the diameter was increased to 6.0 wavelengths, the eigenvalue and moment method results were again nearly the same, except for the small but noticeable difference in magnitudes. Adding some 56 numbers, each from a combination of Bessel functions accurate to only 5 decimal places could contribute significantly to this difference. To determine the degree to which the results are correct, one recommended approach would be to bound the error due to truncation. A second approach would be to run these programs on a system which has a longer word length, so that the machine generated errors will be reduced. An additional reason for going to a larger machine would hopefully be to decrease the run time required by each program and to have more memory to work with. More accurate evaluation of the Bessel functions could also lead to better results.

The two methods studied do present and address some of the complexities encountered in trying to solve the problem. A better understanding of this problem could come from also plotting the magnitude and phase of the incident fields, the field in the dielectric, and the scattered field. This would give an indication of the interactions of the fields producing the total field observed. Yet, the fact remains that the point matching method for the structure of 60 wavelength diameter required more memory than is available to the average system user. Some method for retaining the data generated such as on magnetic tape is needed to circumvent the core limitations. The other methods for reducing the size of the matrix such as Lagrange Interpolation

can be tested.

With results for scattering from the circular obstacle of 6.0 wavelength diameter the scatterer could then be modeled as an ellipse and verified by an essentricity of zero. The essentricity could then be allowed to approach one, which would be a model more representative of the radome of concern. The methods developed to reduce the size of the matrix for the circular obstacle could possibly be manipulated for use with the ellipse. The ellipse will present its own problems such as how the cell size should vary around the structure and what effect a very close source (relative to the size of the obstacle) will have. But, these steps should lead to an eventual solution to the secondary lobe seen in the measured antenna patterns.

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Appendix A. The Model and Its Parameters

To model the radome as a dielectric cylinder of circular cross section the radius of curvature of the radome must be determined. Figure A-1 shows the geometry for the model and for the approach to be taken.

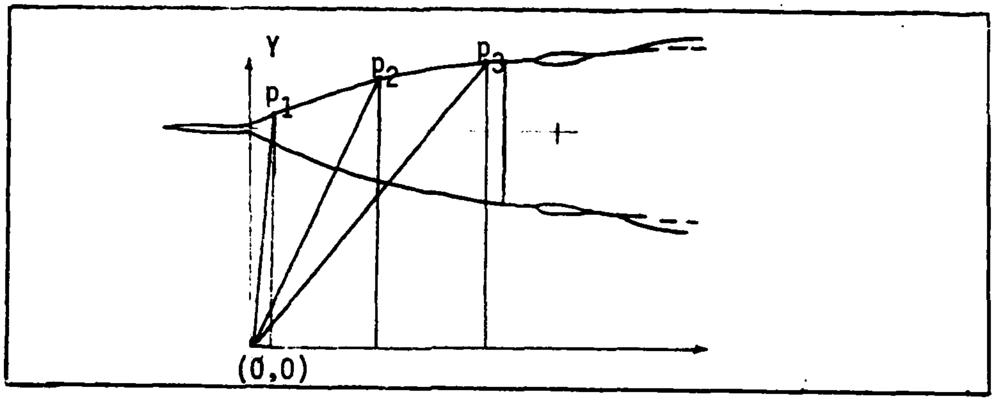


Figure A-1. Model and geometry to determine the radius of curvature of the radome.

A result for the radius of curvature can be obtained using the following procedure.

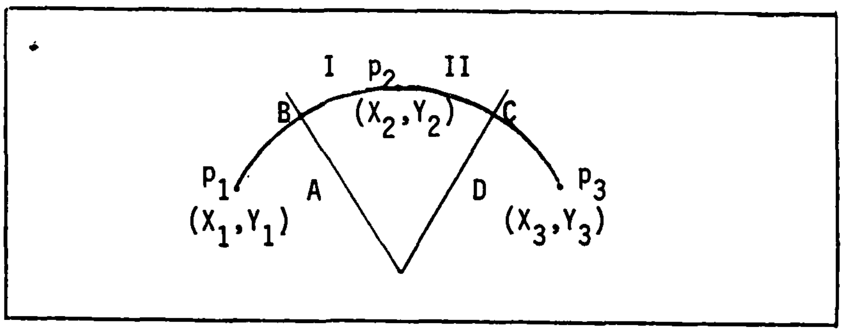


Figure A-2. The geometry to be used in solving for the radius of curvature of a circular arc.

The equation for the arc S from p_1 to p_3 is

$$Y-Y_1 = \frac{Y_2-Y_1}{X_2-X_1} (X-X_1) \tag{A-1}$$

The equation for vector A which is the perpendicular bisector of the arc from p_1 to p_2 is

$$Y_A = -\frac{X_2-X_1}{Y_2-Y_1} X_A + b \tag{A-2}$$

The midpoint of segment I (B to p_2) is

$$X = \frac{X_1+X_2}{2}, \quad Y = \frac{Y_1+Y_2}{2} \tag{A-3}$$

Substituting (A-3) into (A-2), (X,Y) being the point at which vector A and segment I intersect, yields

$$\frac{Y_1+Y_2}{2} = -\frac{X_2-X_1}{Y_2-Y_1} \left(\frac{X_1+X_2}{2}\right) + b \tag{A-4}$$

Solving (A-4) for b results in

$$b = \frac{Y_1+Y_2}{2} + \frac{X_2^2-X_1^2}{2(Y_2-Y_1)} \tag{A-5}$$

Replacing b in (A-2) by (A-5), the equation for vector A is

$$Y_A = -\frac{X_2-X_1}{Y_2-Y_1} X_A + \frac{Y_1+Y_2}{2} + \frac{X_2^2-X_1^2}{2(Y_2-Y_1)} \tag{A-6}$$

Similarly, the equation for vector D is

$$Y_D = -\frac{X_3-X_2}{Y_3-Y_2} X_D + \frac{Y_2+Y_3}{2} + \frac{X_3^2-X_2^2}{2(Y_3-Y_2)} \tag{A-7}$$

Solving (A-6) and (A-7) simultaneously for the point of intersection (h,k), ($Y_A = Y_D = k$, $X_A = X_D = h$), yields

$$-\frac{X_2-X_1}{Y_2-Y_1} h + \frac{Y_1+Y_2}{2} + \frac{X_2^2-X_1^2}{2(Y_2-Y_1)} = -\frac{X_3-X_2}{Y_3-Y_2} h + \frac{Y_2+Y_3}{2} + \frac{X_3^2-X_2^2}{2(Y_3-Y_2)}$$

or

$$h = \left[\frac{Y_2+Y_3}{2} + \frac{X_3^2-X_2^2}{2(Y_3-Y_2)} - \frac{Y_1+Y_2}{2} - \frac{X_2^2-X_1^2}{2(Y_2-Y_1)}\right] \left[\frac{X_3-X_2}{Y_3-Y_2} - \frac{X_2-X_1}{Y_2-Y_1}\right]^{-1}$$

and simplifying,

$$h = \frac{1}{2}\left[\frac{(Y_3-Y_1)(Y_3-Y_2)(Y_2-Y_1)+(X_3{}^2-X_2{}^2)(Y_2-Y_1)-(X_2{}^2-X_1{}^2)(Y_3-Y_2)}{(X_3-X_2)(Y_2-Y_1)-(X_2-X_1)(Y_3-Y_2)}\right] \tag{A-8}$$

Knowing h from (A-8), k can be calculated from (E-6) where

$$k = -\frac{X_2-X_1}{Y_2-Y_1}h + \frac{Y_1+Y_2}{2} - \frac{X_2{}^2-X_1{}^2}{2(Y_2-Y_1)} \tag{A-9}$$

The equation for the radius of a circle is

$$r = [(X_1-h)^2 + (Y_1-k)^2]^{1/2} \tag{A-10}$$

Figure A-1 shows the model of the radome as a circular cylinder. The radius of curvature of the cylinder can be arrived at using (A-8), (A-9), and (A-10).

Appendix B. Derivation of the Eigenvalue Solution Matrix

Given the following representations of the vector electric potentials in the indicated regions

REGION	ELECTRIC VECTOR POTENTIAL		
0	$A_{z_{o,n}} = F_n H_n^{(2)} (k_o\rho) \cos(n\phi)$	(a)	
I	$A_{z_{I,n}} = [A_n J_n(k_o\rho)+B_n N_n(k_o\rho)] \cos(n\phi)$	(b)	(B.1)
II	$A_{z_{II,n}} = [C_n J_n(k_2\rho)+D_n N_n(k_2\rho)] \cos(n\phi)$	(c)	
III	$A_{z_{III,n}} = E_n J_n(k_o\rho) \cos(n\phi)$	(d)	

the following boundary conditions (B.C.'s)

$$\hat{n} \times \overline{E}_{i,n} = \hat{n} \times \overline{E}_{(i+1),n}\ ,\quad i=0,I,II \ @\ \rho = a,b,c \text{ respectively} \qquad (a)$$

$$\hat{n} \times \overline{H}_{i,n} = \hat{n} \times \overline{H}_{(i+1),n}\ ,\quad i=0,I,II \ @\ \rho = c,b \text{ respectively} \qquad (b) \qquad (B.2)$$

$$\hat{n} \times \overline{H}_{0,n} - \hat{n} \times \overline{H}_{I,n} = \overline{J}_s \qquad @\ \rho = a \qquad (c)$$

and the fact that

$$\overline{E}_{i,n} = E_{z_{i,n}} \hat{z} = j\omega\mu_o A_{z_{i,n}} \qquad (B.3)$$

a matrix can be formed to solve for the six unknowns $A_n \ldots F_n$. Assume no variation with $(\frac{\partial}{\partial z} = 0)$.

From B.C. (B.2.a) and (B.1) three equations result

$$F_n H_n^{(2)}(k_o a) = A_n J_n(k_o a) + B_n N_n(k_o a) \qquad (B.4)$$

$$A_n J_n(k_o C) + B_n N_n(k_o C) = C_n J_n(k_2 C) + D_n N_n(k_2 C) \qquad (B.5)$$

$$C_n J_n(k_2 b) + D_n N_n(k_2 b) = E_n J_n(k_o b) \qquad (B.6)$$

For the case at hand

$$\overline{H} = -\frac{1}{j\omega\mu_o}\left[\frac{\partial E_z}{\partial \rho}\ \hat{\phi}\ \frac{1}{\rho}\ \frac{\partial E_z}{\partial \phi}\ \hat{\rho}\right]$$

$$= -\frac{\partial A_z}{\partial \rho}\ \phi - \frac{1}{\rho}\ \frac{\partial A_z}{\partial \phi}\ \hat{\rho} \qquad \text{(B.7)}$$

In both (B.s.b) and (B.2.C) $\hat{n} = \hat{\rho}$ so that

$$n \times \overline{H} = -\ \frac{\partial A_z}{\partial \rho}\ \hat{z} \qquad \text{(B.8)}$$

From B.C. (B.2.b), (B.1), and (B.8) two more equations result

$$k_o[A_nJ_n'(k_oC)+B_nN_n'(k_oC)] = k_2[C_nJ_n'(k_2C)+D_nN_n'(k_2C)] \qquad \text{(B.9)}$$

$$k_2[A_nJ_n'(k_2b)+B_nN_n'(k_2b)] = k_oE_nJ_n'(k_ob) \qquad \text{(B.10)}$$

The current density is given by

$$\overline{J} = J_z\ \hat{z} = I\delta(\overline{r})\ \hat{z} \qquad \text{(B.11)}$$

where I is the magnitude of the source current and $\delta(\overline{r})$ is the Kronecker delta function. The total current is obtained by integrating over the cross sectional area through which the current density passes

$$I_{Total} = \int_S \overline{J}\ ds = I \qquad \text{(B.12)}$$

Integrating (B.11) over a circular cross section

$$\int_S \delta(\overline{r})\ ds = 1$$

$$= \iint \delta(\overline{r})\ \rho d\rho d\phi \qquad \text{(B.13)}$$

which yields

$$\delta(\overline{r}) = \frac{1}{\rho}\ \delta(\rho)\ \delta(\rho) \qquad \text{(B.14)}$$

At $\rho = a$

$$J_z = \frac{1}{\rho}\ \delta(\rho-a)\delta(\phi) \qquad \text{(B.15)}$$

where $\delta(\rho-a)$ is the Dirac delta function.

From B.C. (B.2.C), (B.11), (B.8), (B.7), and

$$E_{z_i} = \sum_{n=-\infty}^{\infty} - j\omega\mu\, A_{z_i} \tag{B.16}$$

(Ref. 1:198)

$$-k_o \sum_{n=-\infty}^{\infty} \{F_n H_n'^{(2)}(k_o a) - [A_n J_n'(k_o a) + B_n N_n'(k_o a)]\} \cos(n\phi) = J_{s_z} \tag{B.17}$$

The surface current J_{s_z} is obtained by integrating (B.15) over the surface of the source.

$$J_{s_z} = \int \frac{I}{\rho}\, \delta(\rho - a)\, \delta(\phi)\, d\rho$$

$$= \frac{I}{a}\, \delta(\phi) \tag{B.18}$$

The surface current can also be expanded in a complete orthonormal basis set

$$J_{s_z} = -k_o \sum_{n=-\infty}^{\infty} a_n \cos(n\phi) \tag{B.19}$$

Equating (B.18) and (B.19)

$$-k_o \sum_{n=-\infty}^{\infty} a_n \cos(n\phi) = \frac{I}{a}\, \delta(\phi)$$

or,

$$-\frac{I}{k_o a}\, \delta(\phi) = \sum_{n=-\infty}^{\infty} a_n \cos(n\phi) \tag{B.20}$$

To apply the rules of orthogonality, multiply both sides of (B.20) by $\cos(m\phi)$ where m, like n, is an integer and integrate over one entire interval 0 to 2π.

$$\int_o^{2\pi} -\frac{I}{k_o a}\, \delta(\phi)\, d\phi = \int_o^{2\pi} \sum_{n=-\infty}^{\infty} a_n \cos(n\phi) \cos(m\phi)\, d\phi \tag{B.21}$$

which reduces to

$$-\frac{I}{k_o a} = \int_o^{2\pi} [a_m \cos(m\phi) \cos(m\phi) + a_{-m} \cos(-m\phi) \cos(m\phi)\, d\phi$$

$$= \pi a_m + \pi a_{-m} \quad , \; m \neq o \tag{B.22}$$

For $a_m = a_{-m}$, as is the case,

$$a_m = \frac{I}{2\pi k_o a} \tag{B.23}$$

If m = o, (B.22) becomes

$$-\frac{I}{k_o a} = 2\pi a_o$$

or,

$$a_o = -\frac{I}{2\pi a k_o} \tag{B.24}$$

Therefore,

$$a_n = -\frac{I}{2\pi a k_o} \quad \text{for all } n \tag{B.25}$$

The complete orthonormal basis set could also have been written as

$$J_{s_z} = -k_o \sum_{n=o}^{\infty} b_n \cos(n\phi) \tag{B.26}$$

since $\cos(n\phi)$ is an even function of ϕ. In the same way as a_n was determined, b_n can be found to be

$$b_n = -\frac{I\varepsilon_n}{2\pi k_o a} \tag{B.27}$$

where

$$\varepsilon_n = \begin{cases} 1, & n = o \\ 2, & n > o \end{cases} \tag{B.28}$$

Substituting (B.19) and (B.25) into (B.17), the result is

$$-k_o \sum_{n=-\infty}^{\infty} \{F_n H_n{}'^{(2)}(k_o a) - [A_n J_n{}'(k_o a) + B_n N_n{}'(k_o a)]\} \cos(n\phi)$$

$$= -k_o \sum_{n=-\infty}^{\infty} -\frac{I}{2\pi k_o a} \cos(n\phi) \tag{B.29}$$

Looking at each term of (B.29) separately yields the sixth equation with which to solve for the unknown coefficients and is

$$A_n J_n{}'(k_o a) + B_n N_n{}'(k_o a) - F_n H_n^{(2)'}(k_o a) = -\frac{I}{2\pi k_o a} \tag{B.30}$$

where (B.30) is based on summing from minus infinity to infinity. If the

same procedure is followed using (B.26), (B.27), and (B.28), the equation becomes

$$A_nJ_n'(k_oa) + B_nN_n'(k_oa) - F_nH_n^{(2)'}(k_oa) = -\frac{I\varepsilon_n}{2\pi k_oa\varepsilon_n}$$

$$= -\frac{I}{2\pi k_oa} \qquad \text{(B.31)}$$

where (B.31) is based on summing from zero to infinity. The six equations derived can be combined to form one matrix equation which can be solved for the six coefficients $A_n \ldots F_n$.

$$\begin{bmatrix} J_n(k_oa) & N_n(k_oa) & 0 & 0 & 0 & -H_n^{(2)}(k_oa) \\ J_n(k_oc) & N_n(k_oc) & -J_n(k_2c) & -N_n(k_2c) & 0 & 0 \\ 0 & 0 & J_n(k_2b) & N_n(k_2b) & -J_n(k_ob) & 0 \\ k_oJ_n'(k_oc) & k_oN_n'(k_oc) & -k_2J_n'(k_2c) & -k_2N_n'(k_2c) & 0 & 0 \\ 0 & 0 & k_2J_n'(k_2b) & k_2N_n'(k_2b) & -k_oJ_n'(k_ob) & 0 \\ J_n'(k_oa) & N_n'(k_oa) & 0 & 0 & 0 & -H_n^{(2)'}(k_oa) \end{bmatrix} \times \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \\ E_n \\ F_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-I}{2\pi k_oa} \end{bmatrix}$$

(B.32)

Appendix C. Representing the Distance to the Far Zone Observation Point

The distance from the source point to the observation point in the far zone, ρ_n, is shown in Figure C-1. The stipulation that must be made is that the distance to the observation point, ρ_o, is much much greater than the distance to the source point, ρ. With this condition met, it can be assumed that for an observation point out at infinity the vectors $\bar{\rho}_o$ and ρ_n become almost parallel and therefore $\bar{\rho}_n$ $\bar{\rho}_n$' (see Figure C-1.).

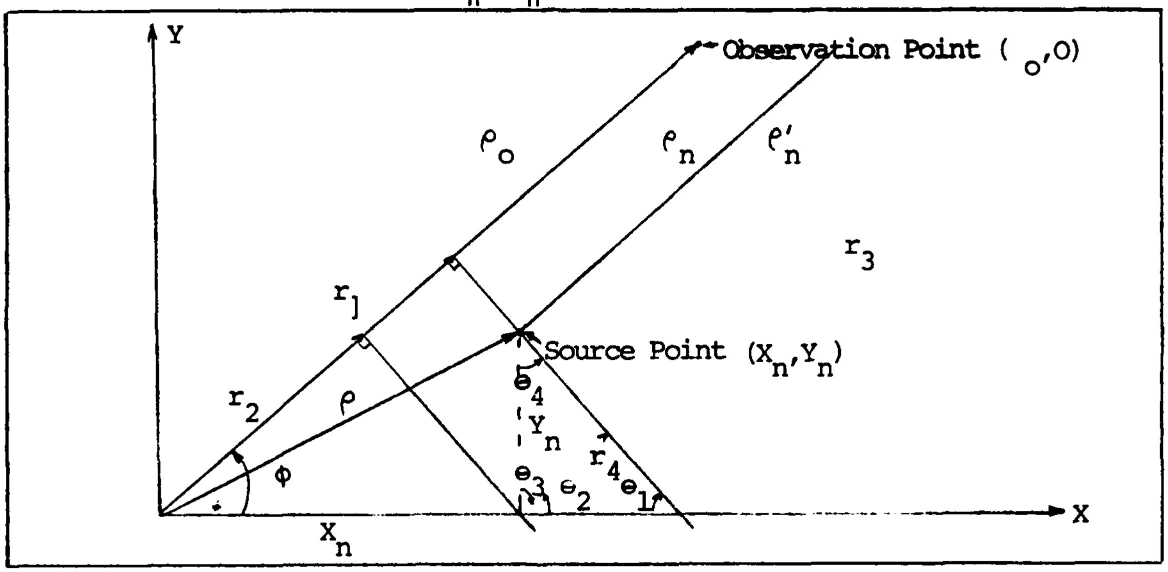


Figure C-1. Geometry for the distance to the observation point in the far field from the source point.

For r_3 parallel to ρ_o angle $\theta_2 = \phi$ and $\theta_3 = 90 - \phi$ which implies that $\theta_4 = \phi$. The geometry is such that $r_4 = r$, and

$$r_1 = r_4 = Y_n \sin\phi \tag{C-1}$$

Also,

$$r_2 = X_n \cos\phi \tag{C-2}$$

Using (C-1) and (C-2), it is obvious that

$$\rho_n \sim \rho_n' = \rho_o \; X_n \cos\phi - Y_n \sin\phi \tag{C-3}$$

Appendix D. Integration of the Hankel Function over a Circular Area

Using the moment method to evaluate the total field due to a dielectric cylindrical shell or circular cross section in the presence of a parallel filament line source, the integral in (D-1) must be evaluated (following Richmond (Ref. 23:336)).

$$jk^2/4 \iint_{\text{cell } n} H_0^{(2)}(k\rho)\ \rho'\ d\rho'\ d\phi' \qquad \text{(D-1)}$$

where

$$\rho = |\rho-\rho'| \qquad \text{(D-2)}$$

see Figure D-1.

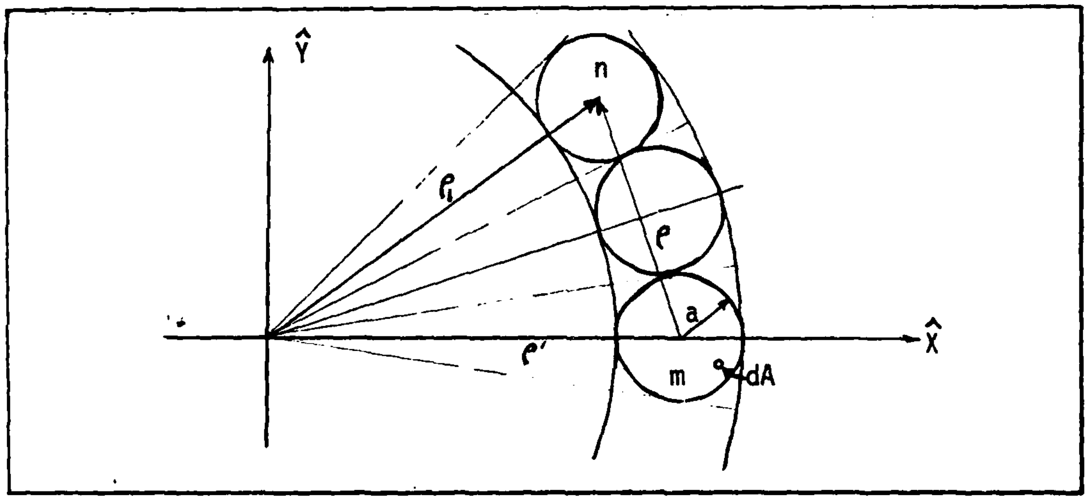


Figure D-1. Division of the obstacle into circular cells.

A singularity exists in evaluating (D-1) when the argument becomes zero or when the source and observation point are one and the same. To evaluate (D-1) for this singularity the integration becomes

$$jk^2/4 \int_0^{2\pi} \int_0^a H_0^{(2)}(k\rho)\ \rho'\ d\rho'\ d\phi' \qquad \text{(D-3)}$$

where

$$H_o^{(2)}(k\rho) = J_o(k\rho) - j\, N_o(k\rho),\ \rho = \rho' \qquad \text{(D-4)}$$

for a reference point at the center of cell n. Substituting (D-4) into (D-3)

$$jk^2/_4 \int_o^{2\pi} \int_o^a H_o^{(2)}(k\rho')\, \rho'\, d\rho'\, d\phi' = jk^2/_4 \{\int_o^{2\pi} \int_o^a \rho' J_o(k\rho')d\rho'\, d\phi'$$

$$- j \int_o^{2\pi} \int_o^a \rho' N_o(k\rho')d\rho' d\phi \qquad \text{(D-5)}$$

Two equations from Abramowitz (Ref. 25:484) 11.3.20 and 11.3.24 are

$$\int_o^z t^\upsilon\, J_{\upsilon-1}(t)dt = z^\upsilon J_\upsilon(z) \qquad \text{(D-6)}$$

$$\int_o^z t^\upsilon\, N_{\upsilon-1}(t)dt = z^\upsilon N_\upsilon(z) + \frac{2^\upsilon(\upsilon)}{\pi} \qquad \text{(D-7)}$$

Splitting (D-5) into two parts and substituting (D-6) and (D-7) in, the following steps result:

$$\upsilon = 1$$

$$\int_o^{2\pi} \int_o^a \rho'\, J_o(k\rho')d\rho'\, d\phi' = 2\pi \int_o^a \rho'\, J_o(k\rho')d\rho'$$

Let

$$t = k\rho' \qquad dt = kd\rho'$$

for

$$\rho' = o \qquad t = o$$

$$\rho' = a \qquad t = ka$$

$$2\pi \int_o^a \rho'\, J_o(k\rho')d\rho' = \frac{2}{k^2} \int_o^{ka} t\, J_o(t)dt$$

$$= \frac{2\pi}{k^2} (ka)\, J_1(ka) \qquad \text{(D-8)}$$

And, as above

$$\int_o^{2\pi} \int_o^a \rho'\, N_o(k\rho')d\rho'\, d\phi' = \frac{2\pi}{k^2} \int_o^{ka} t\, N_o(t)dt$$

$$= \frac{2\pi}{k^2} \{ka\, N_1(ka) + \frac{2\,(\Gamma)}{\pi}\} \qquad \text{(D-9)}$$

Substituting (D-8) and (D-9) into (D-5), the result is

$$jk^2/_4 \{\frac{2\pi}{k^2}[(ka)\ J_1(ka)] - j\ \frac{2\pi}{k^2}[(ka)\ N_1(ka) + \frac{2}{\pi}]\}$$

which reduces to

$$j/_2 \{\pi ka\ J_1(ka) - j\ \pi ka\ N_1(ka) - j\ 2\}$$

or

$$jk^2/_4 \int_0^{2\pi} \int_0^a H_0^{(2)}(k\rho')\rho'\ d\rho'\ d\phi' = j/_2(\pi ka\ H_1^{(2)}(ka) - j\ 2) \qquad \text{(D-10)}$$

for

$$m = n.$$

Appendix E. Results

Contained in this appendix are the results generated by the program implementing the eigenvalue solution method and the program implementing the moment method solution.

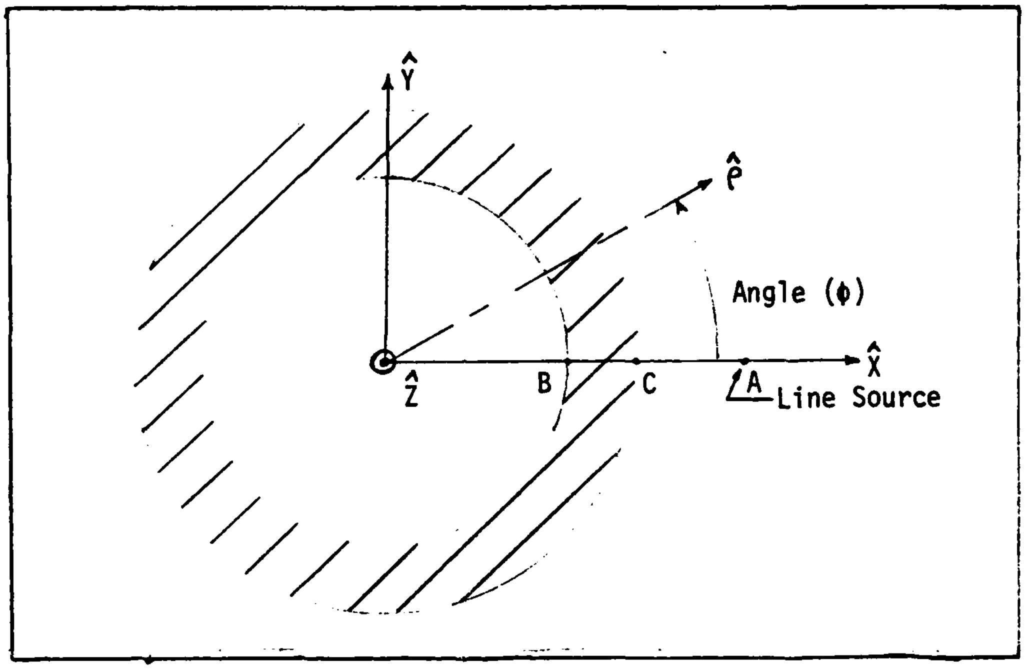


Figure E-1. The geometry used in the computer programs to produce the results contained within this appendix.

For all the results

Permeability $\mu_o = \mu$

Dielectric Constant $\epsilon_R = 4.0$

Loss Tangent $\tan\theta = 0.0$

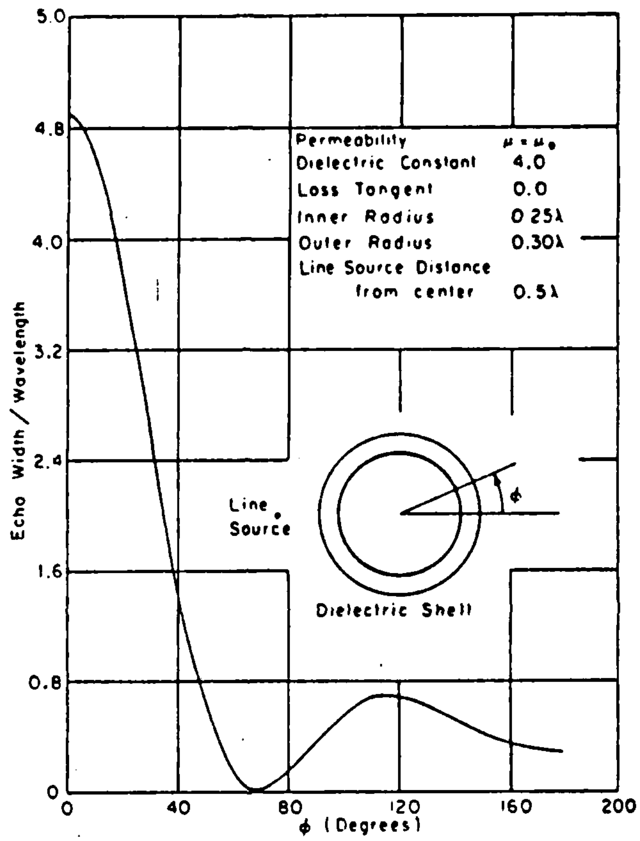


Figure E-2. Scattering pattern of a circular dielectric shell in the presence of a nearby parallel line source, calculated with the in-egral-equation technique.

Reproduced fran Richmond (Ref. 23:338)

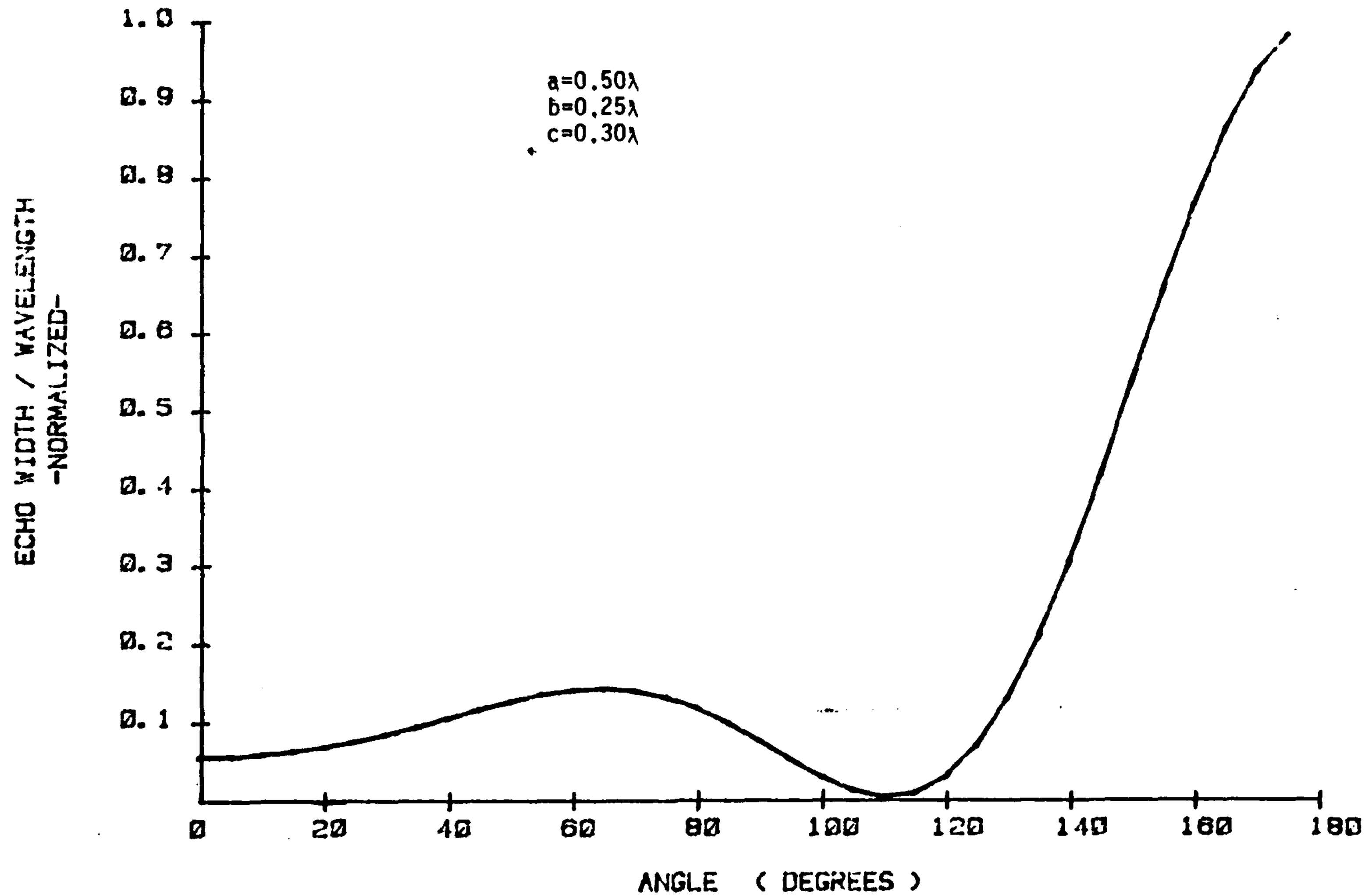


Figure E-3. Scattering pattern using the eigenvalue solution method.

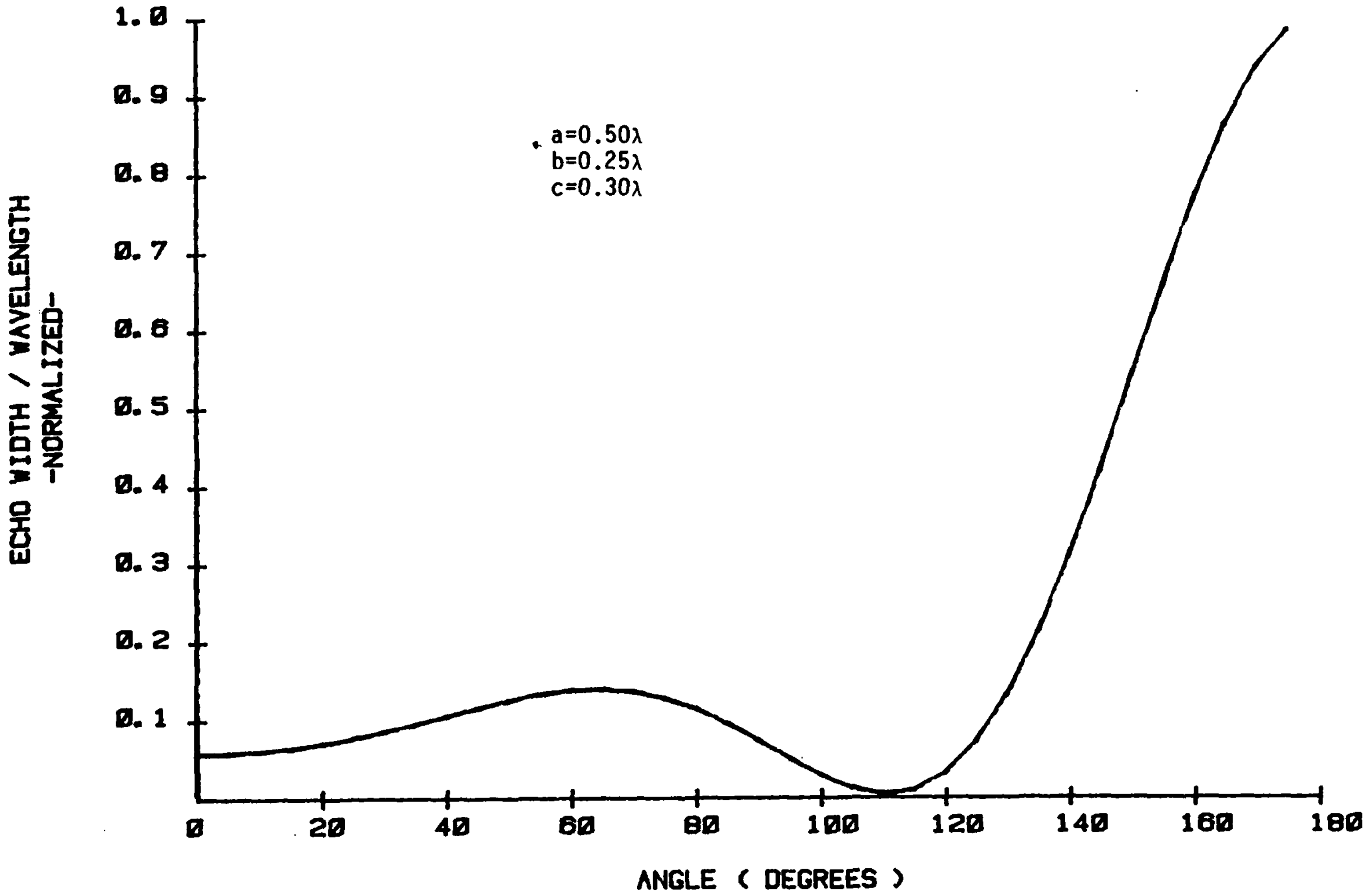


Figure E-4. Scattering pattern using the moment method solution.

TABLE E-I.

Comparison of the Results from the Eigenvalue and Moment Method Programs to Richmond's Results.

For the three sets of results:

Permeability	$\mu = \mu_o$
Dielectric Constant	4.0°
Loss Tangent	0.0
Inner Radius	0.25λ
Outer Radius	0.30λ
Line Source Distance from Center	0.5λ

Echo Width/Wavelength			
Angle	Richmond	Eigenvalue	Moment Method (37 cells)
0	0.284	0.282	0.289
70	0.697	0.671	0.720
110	0.026	0.022	0.022
180	4.90	4.91	5.11

NOTE: Richmond (Ref. 23:338) placed the line source at 180° while the source was at zero degrees for the computer programs. Flipping Richmond's results over allows the comparison above.

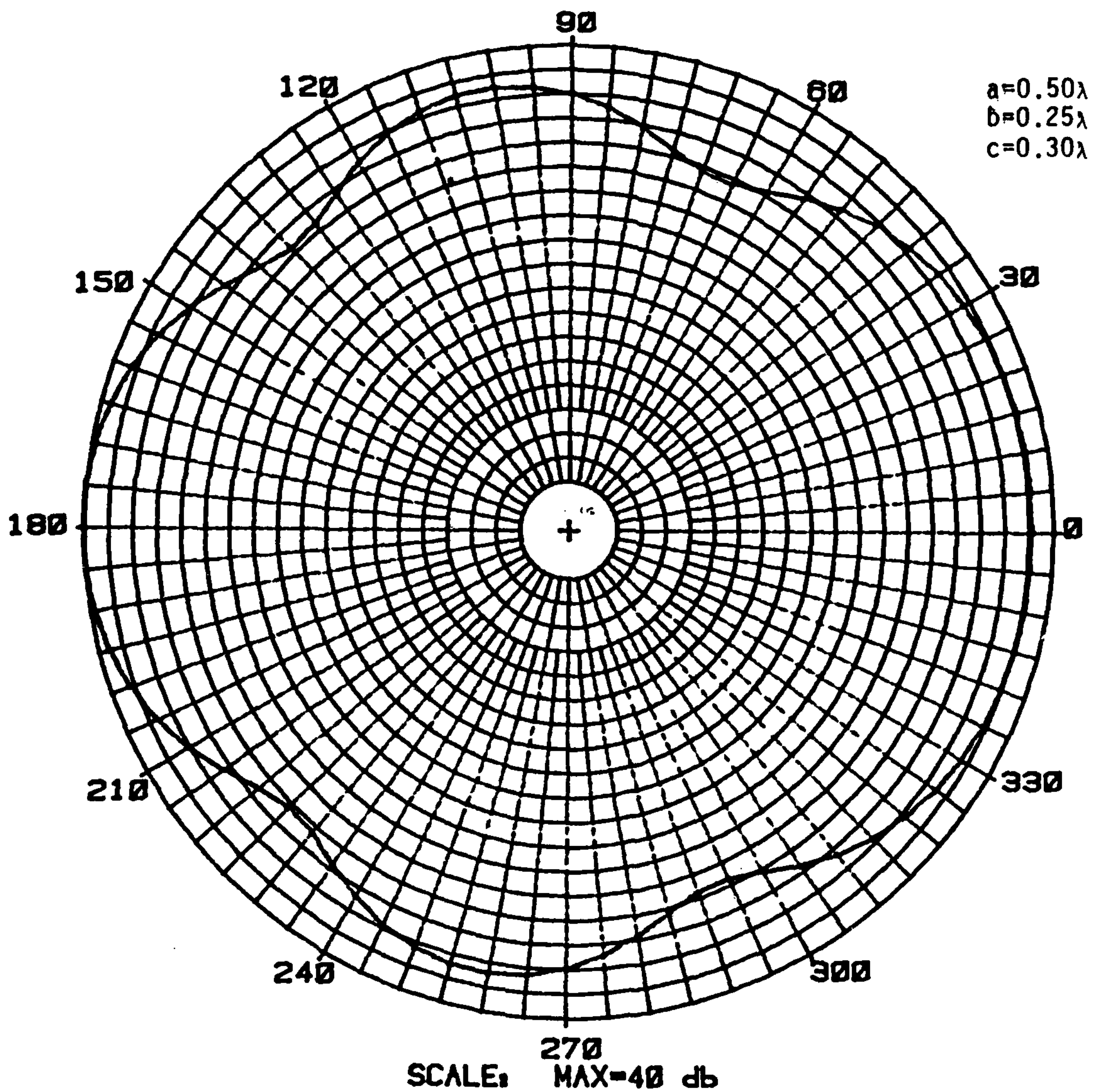


Figure E-5. Total electric field using the eigenvalue solution method.

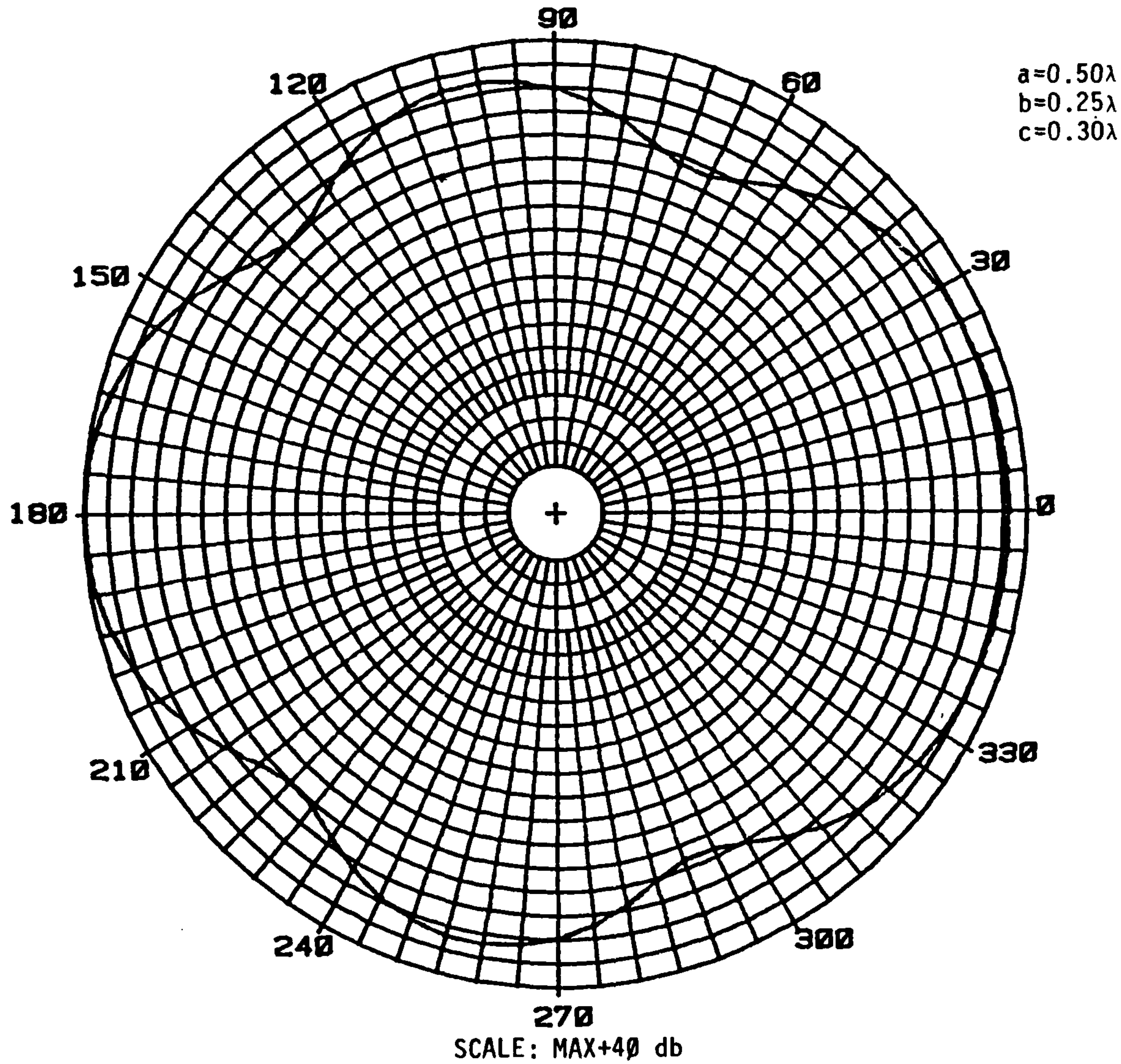


Figure E-6. Total electric field using the moment method solution.

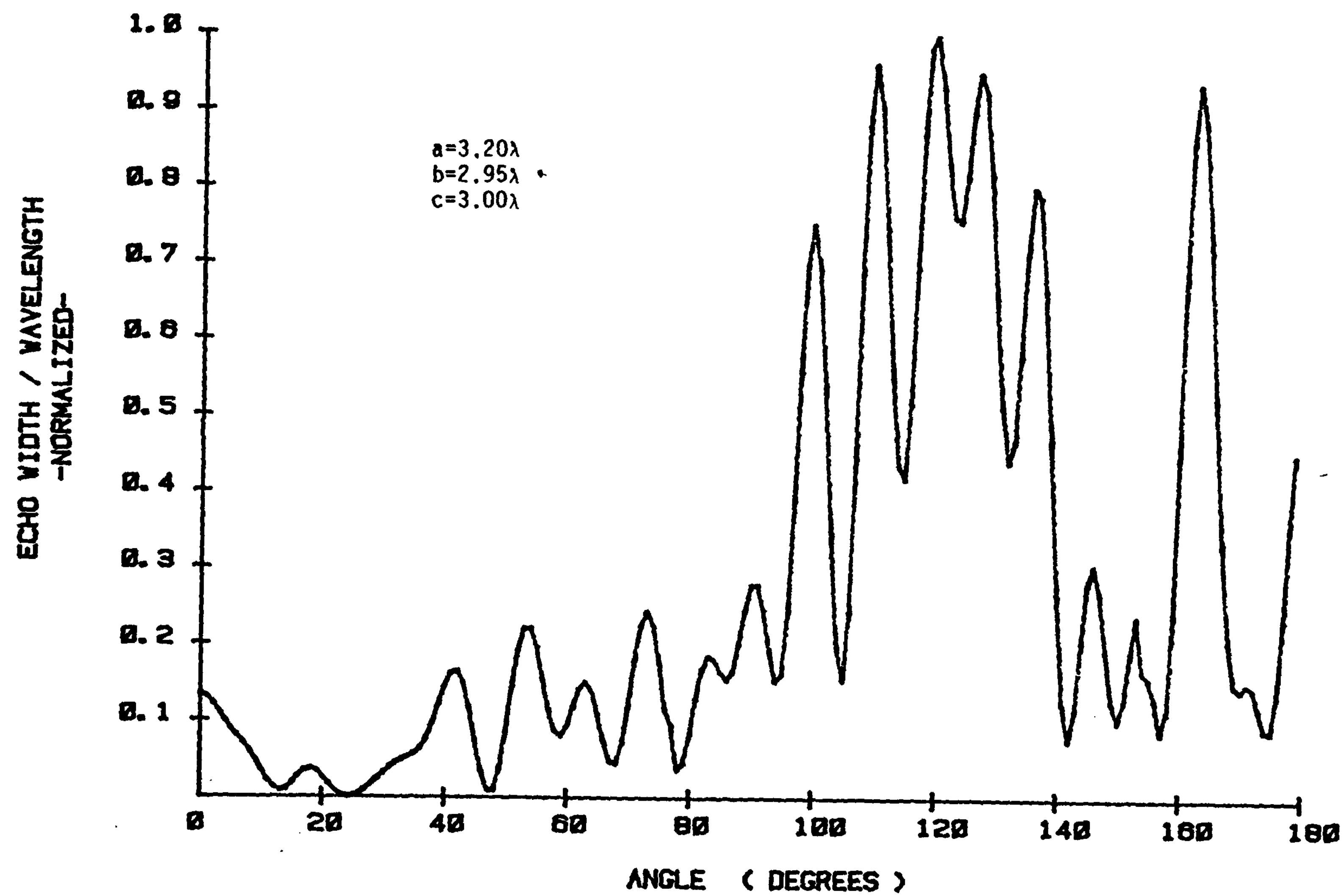


Figure E-7. Scattering pattern using the eigenvalue solution method.

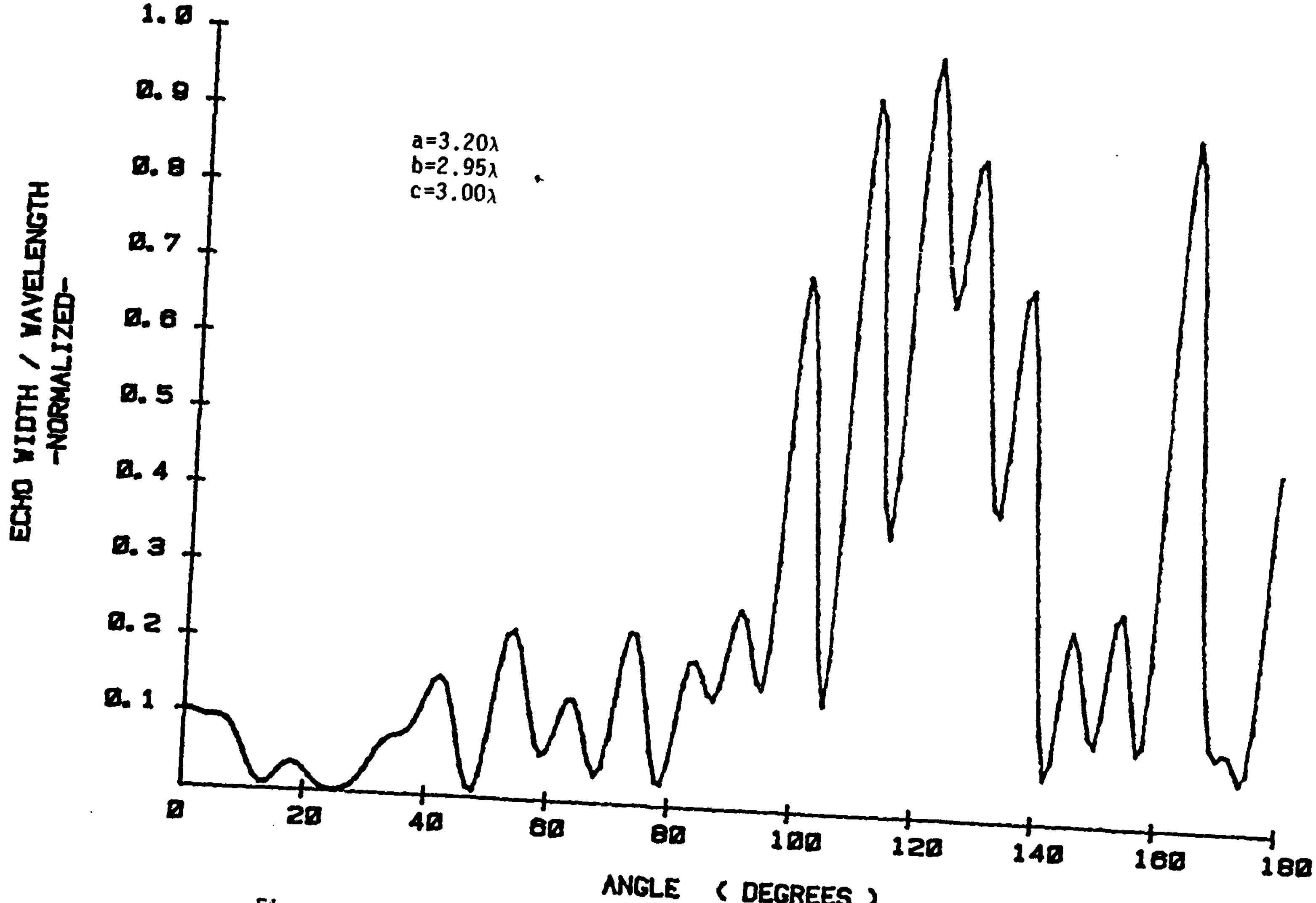


Figure E-8. Scattering pattern using the moment method solution.

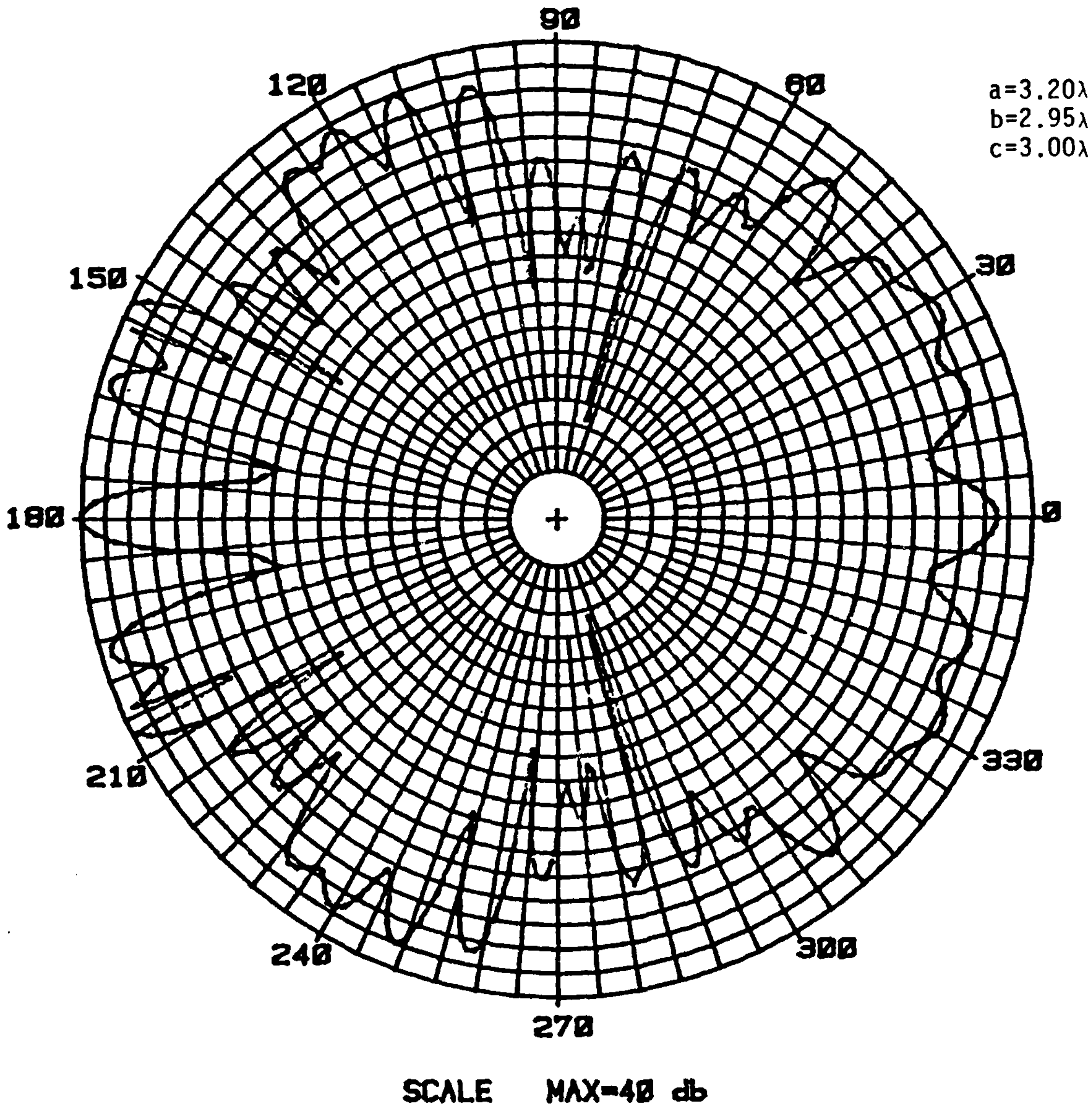
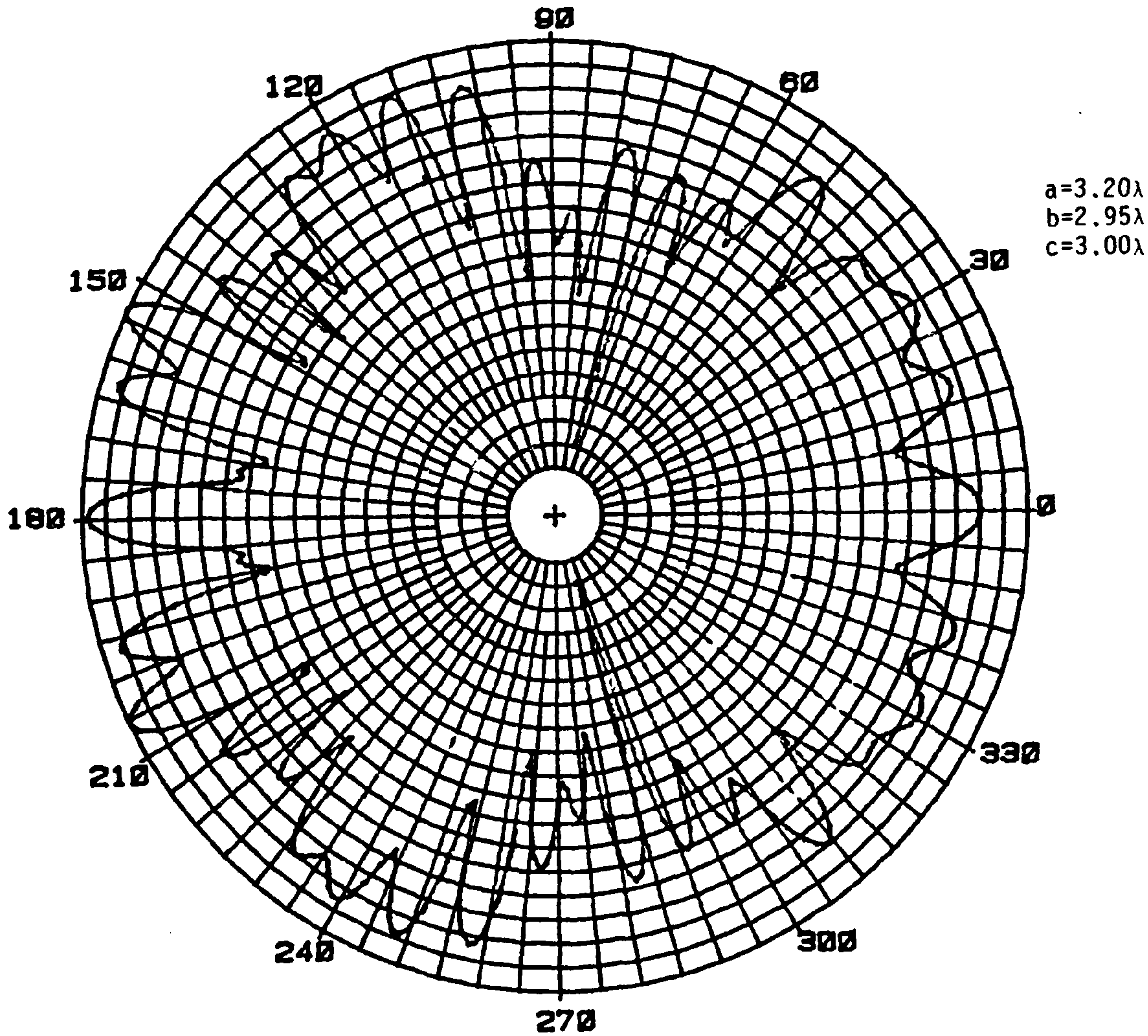
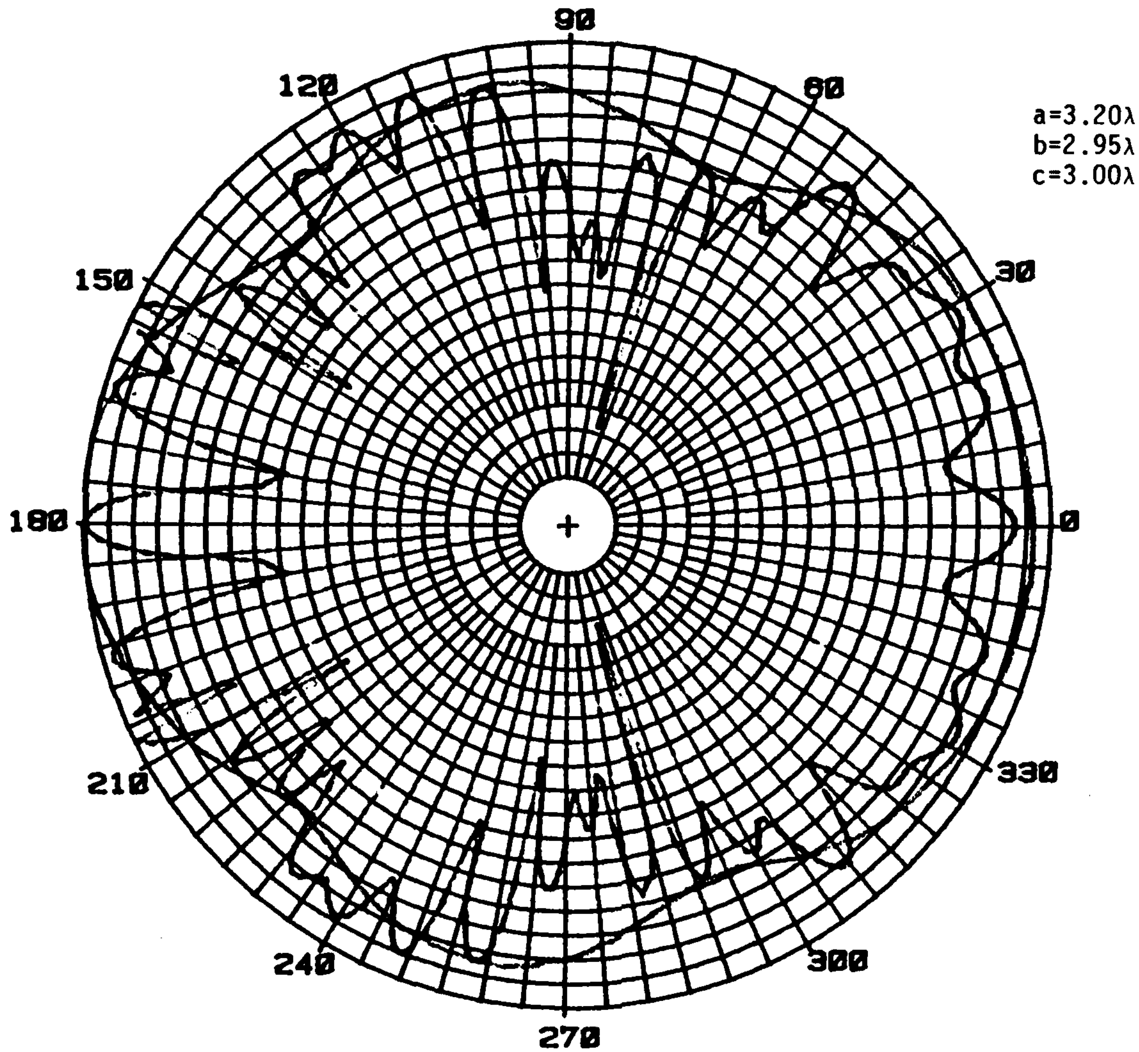


Figure E-9. Total electric field using the eigenvalue solution method.



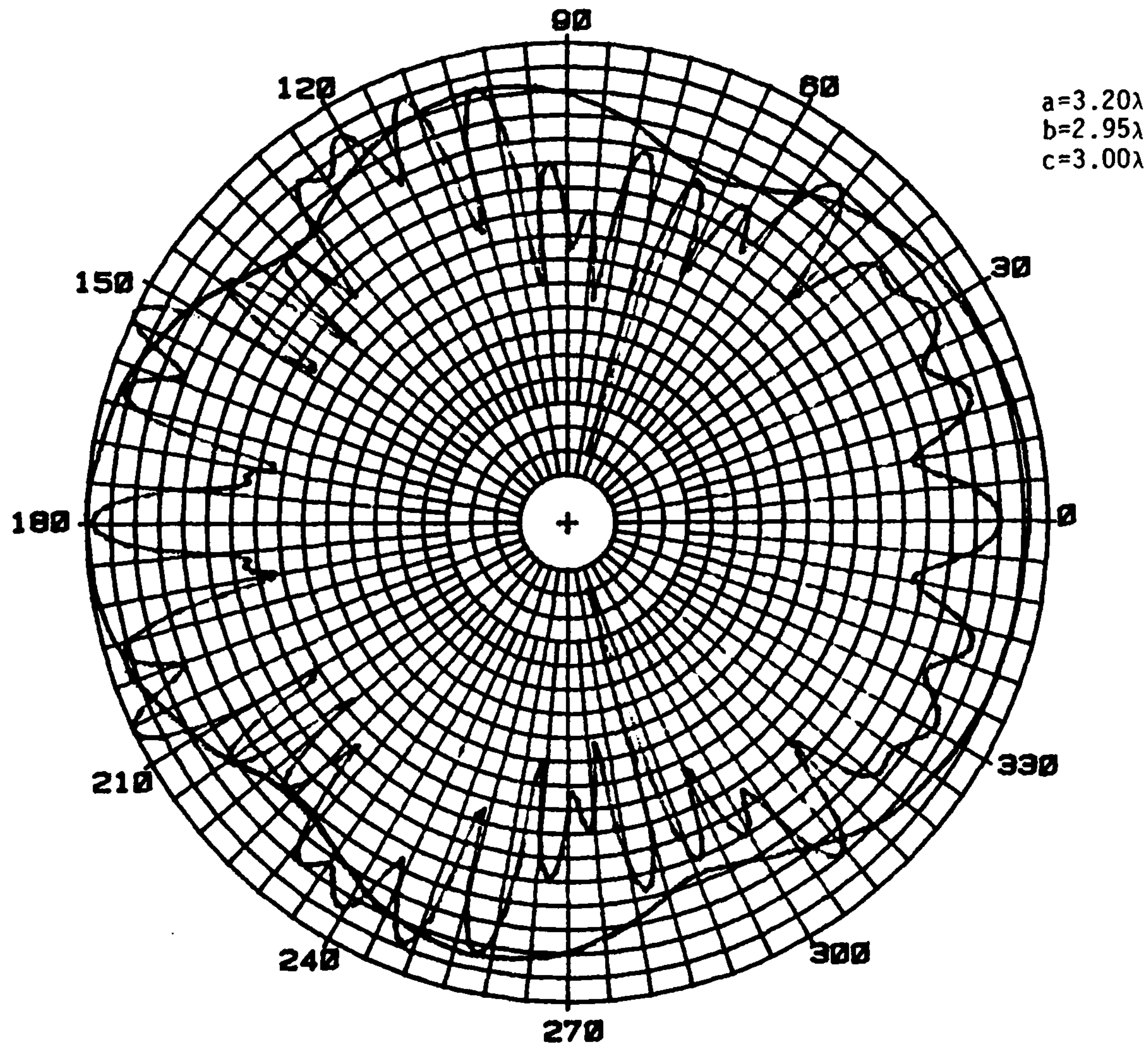
SCALE: MAX=40 db

Figure E-10. Total electric field using moment method solution.



SCALE : MAX=40 db

Figure E-11. Overlaying Figure E-5 on Figure E-9 for the total field using the eigenvalue solution method.



SCALE: MAX-40 db

Figure E-12. Overlaying Figure E-6 on Figure E-10 for the total field using the moment method solution.

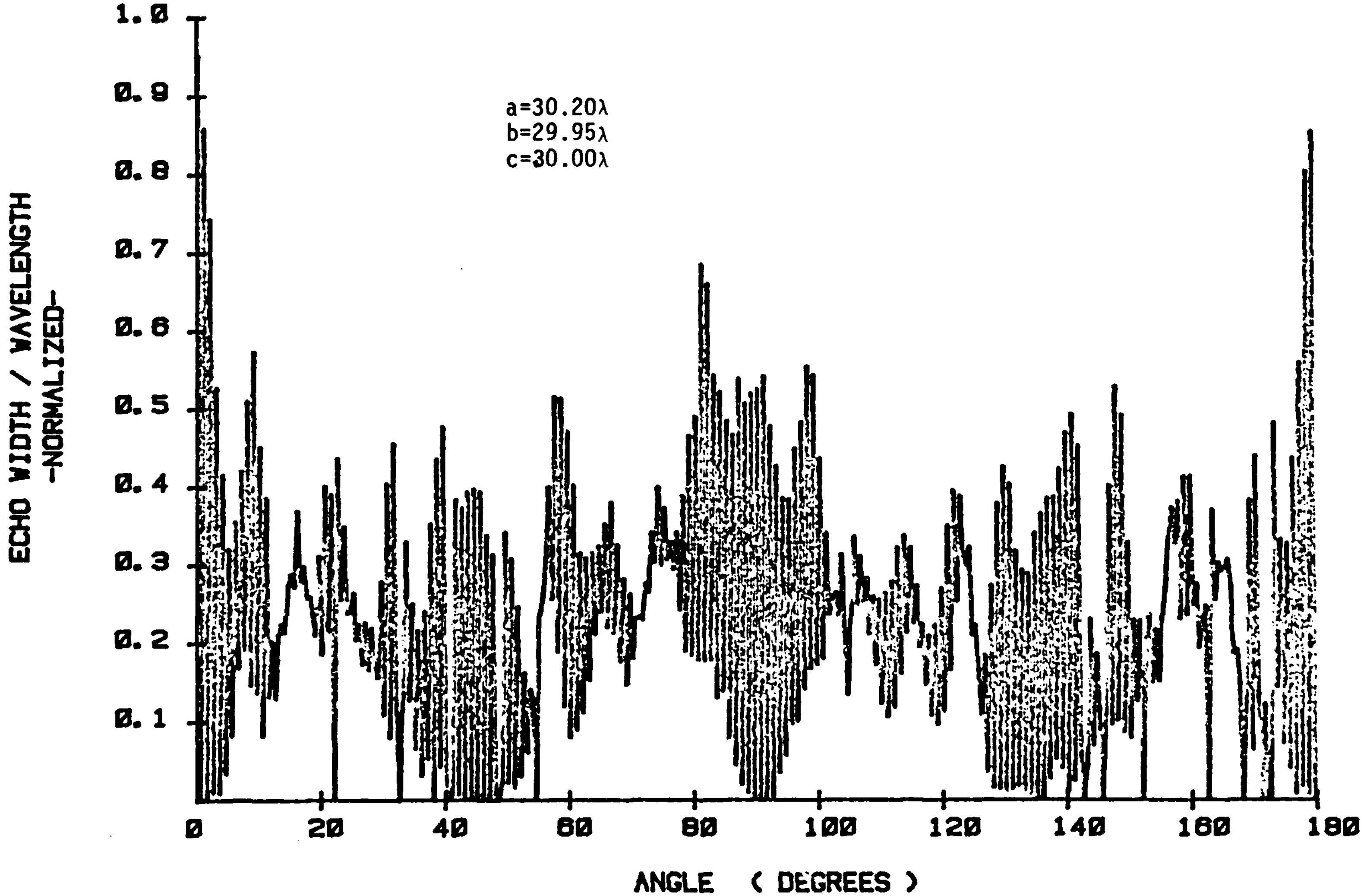


Figure E-13. Scattering pattern using the eigenvalue solution method.

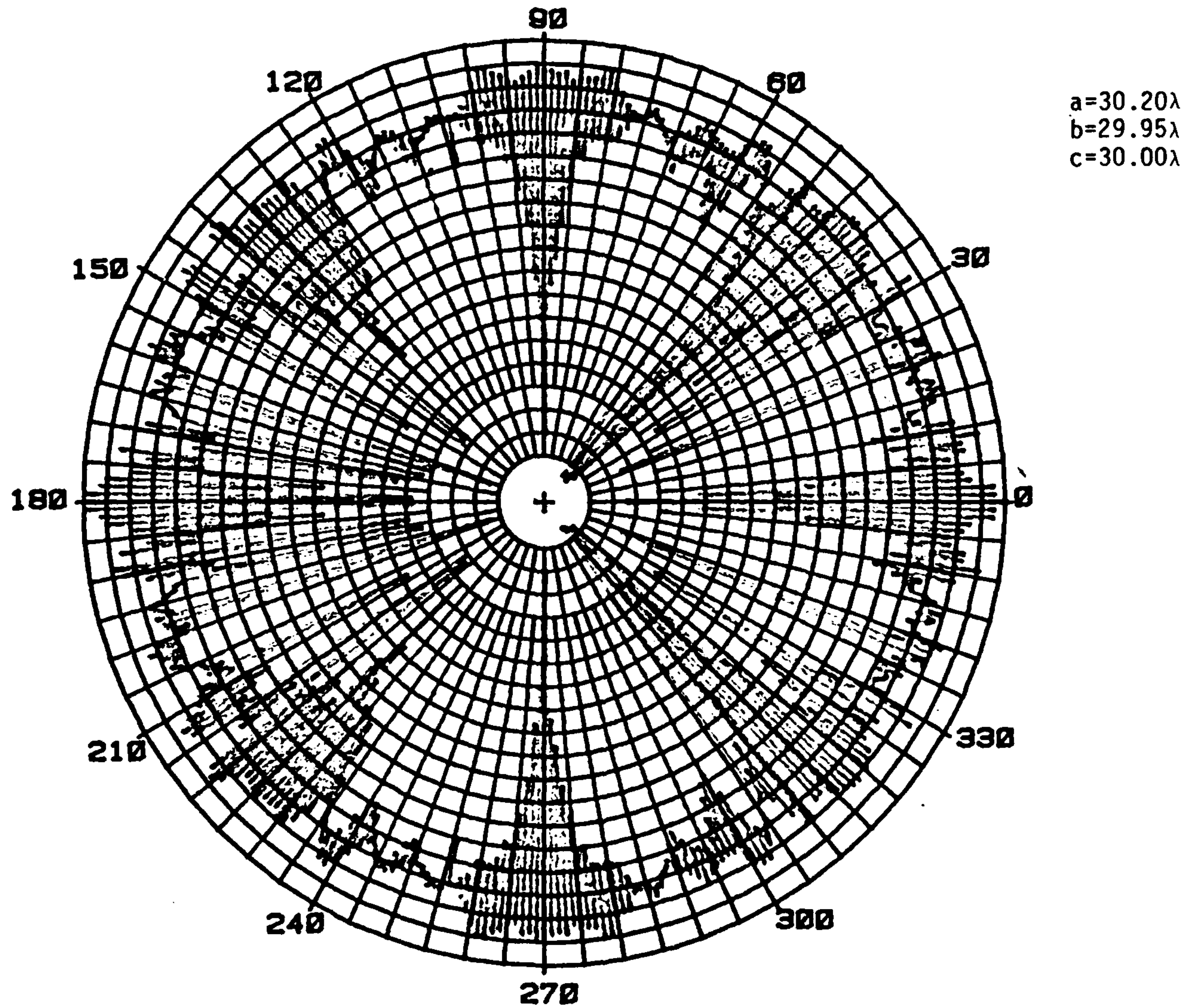


Figure E-14. Total electric field using the eigenvalue solution method.

Appendix F. Computer Program for Eigenvalue Solution Method

This appendix contains the computer program which implements the eigenvalue solution method. The main program, RADMI, the matrix inversion routine, CLSKY, and the routine to calculate the value of Bessel functions of the first and second kind and their derivatives, BESEL, are written in FORTRAN 4. The computer system used was the Hewlett-Packard 21MX M series minicomputer.

```
1I T=00004 IS ON CR00039 USING 00029 BLKS R=0000

 FTN4,L
 $EMA(CMBLK,0)
 C *************************
       PROGRAM RADM1
 C *************************
 C
 C                                    ====================
 C                                     ROBERT K. SCHNEIDER
 C                                    ====================
 C
 C * THIS PROGRAM EVALUATES THE EIGENVALUE SOLTION FOR THE RADIATED FIELD
 C     OUTSIDE AN INFINITELY LONG DIELECTRIC CYLINDRICAL SHELL OF INNER
 C     RADIUS B AND OUTER RADIUS C.  THE INCIDENT FIELD HAS AS ITS SOURCE
 C     AN INFINITE LINE CURRENT CLOSE TO, BUT EXTERNAL TO, THE CYLINDER
 C     AND AT AN ANGLE PHI-PRIME EQUAL TO ZERO.
 C
 C * A MATRIX EIGENVALUE SOLUTION HAS BEEN FORMULATED LEAVING THE COEFFI-
 C     CIENTS OF THE FIELD TO BE SOLVED FOR.  AN ALGORITHM BY PRESCOTT D.
 C     CROUT IS USED TO SOLVE FOR THESE COEFFICIENTS.
 C
 C * THE FAR FIELD IS GIVEN BY :
 C
 C           Ezon = -jwuFn TIMES THE HANKEL FUNCTION OF THE 2nd KIND
 C                  OF ARGUEMENT K0*RAU AND ORDER n.
 C
 C     THE TOTAL FIELD IS A SUM ON n OF THE ABOVE.    A CONVERGENCE OF THIS
 C     SUM IS DETERMINED BY THIS PROGRAM.  ALSO, SINCE THE ARGUEMENT OF THE
 C     ABOVE HANKEL FUNCTION IS >> n, THE APPROPRIATE ASYMPTOTIC EXPANSION
 C     IS USED.
 C
 C * EPSLN IS A VALUE INPUT BY THE USER FOR USE IN DETERMINING CONVERGENCE.
 C
 C * K0 = WAVE NUMBER IN FREE SPACE
 C
 C * K2 = WAVE NUMBER IN DIELECTRIC
 C
       INTEGER ESTIME
 C
       REAL K0,K2,MAX,NRMFLD,HPHI,J0
 C
       COMPLEX HN20A,HNP20A,SUMEZ,A(6,7),F(6),FN(1001),D11,D12,
      1        SUMN,SUMP,SUM,EZON,D5
 C
       DIMENSION IBUFR(16),J2(10)
 C
       COMMON /CMBLK/ A,F,ECHWPW(3600),DGRENC,SCRPEZ(3600),
      1               DBPRPT(3600)
 C
       DATA PI/3.14159265/
       DATA XCR/4.875/,YCR/3.875/,RDS/2.748/
 C
 C * SET UP CALL FOR INPUT FROM DISK FILE "DATA1"
 C
       DATA IBUFR/2*0,2HDA,2HTA,2H1 ,3*0,221B,7*0/
 C
       CALL SPOPN(IBUFR,ISLU)
 C
       CALL EXEC(22,1)
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C
      READ(ISLU,*) AA,B,C,CURENT,R,EPSLN,FREQ,PERM,DGRENC,EPSLNR,
     1              ESTIME,N,ISKIP
C
C * SET DEFAULT FOR ESTIME
C
       IF(ESTIME .EQ. 0) ESTIME=500
C
      ZERO=0.0
C
C * CALCULATE THE WAVE NUMBERS
C
       K0=2.0*PI*FREQ/3.E8
       K2=K0*SQRT(EPSLNR)
C
C * COMPUTE ARGUEMENTS FOR BESEL FUNCTION CALLS
C
       X1=K0*AA
       X2=K0*B
       X3=K0*C
       X4=K2*B
       X5=K2*C
C
C * LOOP ESTINE TIMES TO DETERMINE THE EIGENVALUE Fn FOR EACH ORDER n.
C
       K=0
C
      DO 100 L=-ESTIME,ESTIME
         K=K+1
         EPSLNN=1.0
C        LM1=L-1
         LM1=L
C
            CALL BESEL(LM1,X1,BB,Y,BP,YP)
          HN20A=CMPLX(BB,-Y)
         HNP20A=CMPLX(BP,-YP)
        A(1,1)=CMPLX(BB,ZERO)
        A(1,2)=CMPLX(Y,ZERO)
        A(1,3)=(0.,0.)
        A(1,4)=(0.,0.)
        A(1,5)=(0.,0.)
        A(1,6)=-HN20A
        A(1,7)=(0.,0.)
CC
        A(6,1)=CMPLX(BP,ZERO)
        A(6,2)=CMPLX(YP,ZERO)
        A(6,3)=(0.,0.)
        A(6,4)=(0.,0.)
        A(6,5)=(0.,0.)
        A(6,6)=-HNP20A
            IF(LM1 .EQ. 0) EPSLNN=0.5
          EQLMT=(CURENT*EPSLNN)/(PI*AA*K0)
        A(6,7)=CMPLX(EQLMT,ZERO)
C
            CALL BESEL(LM1,X2,BB,Y,BP,YP)
          DUM1=-K0*BP
        A(3,5)=CMPLX(-BB,ZERO)
CC
        A(5,5)=CMPLX(DUM1,ZERO)
C
```

```
          CALL BESEL(LM1,X3,BB,Y,BP,YP)
         DUM1=K0*BP
         DUM2=K0*YP
        A(2,1)=CMPLX(BB,ZERO)
        A(2,2)=CMPLX(Y,ZERO)
CC
        A(4,1)=CMPLX(DUM1,ZERO)
        A(4,2)=CMPLX(DUM2,ZERO)
C
          CALL BESEL(LM1,X4,BB,Y,BP,YP)
         DUM1=K2*BP
         DUM2=K2*YP
        A(3,1)=(0.,0.)
        A(3,2)=(0.,0.)
        A(3,3)=CMPLX(BB,ZERO)
        A(3,4)=CMPLX(Y,ZERO)
        A(3,6)=(0.,0.)
        A(3,7)=(0.,0.)
CC
        A(5,1)=(0.,0.)
        A(5,2)=(0.,0.)
        A(5,3)=CMPLX(DUM1,ZERO)
        A(5,4)=CMPLX(DUM2,ZERO)
        A(5,6)=(0.,0.)
        A(5,7)=(0.,0.)
C
          CALL BESEL(LM1,X5,BB,Y,BP,YP)
         DUM1=-K2*BP
         DUM2=-K2*YP
        A(2,3)=CMPLX(-BB,ZERO)
        A(2,4)=CMPLX(-Y,ZERO)
        A(2,5)=(0.,0.)
        A(2,6)=(0.,0.)
        A(2,7)=(0.,0.)
CC
        A(4,3)=CMPLX(DUM1,ZERO)
        A(4,4)=CMPLX(DUM2,ZERO)
        A(4,5)=(0.,0.)
        A(4,6)=(0.,0.)
        A(4,7)=(0.,0.)
C
C * NOW THAT THE MATRIX A IS FORMED, CALL CLSKY TO SOLVE FOR THE
C    EIGENVALUES.
C
         M=N+1
        CALL CLSKY(N,M)
C
C * PULL OUT Fn FROM ARRAY F
C
         IF(K .NE. 10) GO TO 17
C       DO 19 I=1,6
C          WRITE(6,18) F(I),L,I
18       FORMAT(1X,"F= ",1E15.8,2X,E15.8,5X,I4,5X,I4,/)
C19      CONTINUE
C        WRITE(6,70)
         K=0
C
17      FN(L+ESTIME+1)=F(6)
C
C        DO 60 M=1,6
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```
C            WRITE(6,50) F(M),M,L
50          FORMAT(1X,"F=",2X,1E15.8,2X,E15.8,5X,I4,5X,I4)
C60          CONTINUE
C            WRITE(6,70)
70          FORMAT(1X,//)
C
100       CONTINUE
C
           WRITE(1,200)
200       FORMAT(1X,"FN EIGENVALUES CALCULATED.")
C
C * CALCULATE THE ECHO WIDTH PER WAVELENGTH RELATIVE TO THE INCIDENT
C    FIELD AT THE CENTER OF THE OBSTACLE
C
            SFLDMX=0.0
            TFLDMX=0.0
            IAVNTM=0.0
            IMAX=0
           NTIMES=IFIX(360./DGRENC)
          DO 700 I=1,NTIMES
             IM1=I-1
             PHI=IM1*DGRENC*PI/180.
             NCNVRG=0
             SUMEZ=(0.,0.)
              SUMM=(0.,0.)
              SUMP=(0.,0.)
C
          DO 550 J=1,ESTIME
             JM1=J-1
             ARGP=JM1*PI/2.
1           D1=SIN(ARGP)
2           D2=COS(ARGP)
              IF(JM1 .EQ. 0) GO TO 540
             ARGM=-ARGP
3           D3=SIN(ARGM)
4           D4=COS(ARGM)
5          SUMM=FN(JM1)*COS(-JM1*PHI)*CMPLX(D3,-D4)
540        SUMP=FN(JM1+ESTIME+1)*COS(JM1*PHI)*CMPLX(D1,-D2)
C
          SUMEZ=SUMEZ+2.*PI*FREQ*PERM*(SUMM+SUMP)
C
6          D10=D2*AIMAG(FN(JM1+ESTIME+1))*COS(JM1*PHI)*2.*PI*FREQ*PERM
               IF(D10 .GT. EPSLN) GO TO 550
                NCNVRG=NCNVRG+1
                J2(NCNVRG)=J
               IF(NCNVRG .LT. 10) GO TO 550
                DO 425 L=2,10
                 IF(J2(L) .NE. J2(L-1)+1) GO TO 427
425             CONTINUE
                 GO TO 510
427            NCNVRG=10-L
                DO 426 L=1,10
                  IF(L .GT. NCNVRG) GO TO 428
                 J2(L)=J2(10-NCNVRG+L)
                  GO TO 426
428              J2(L)=0
426             CONTINUE
550            CONTINUE
C
                WRITE(6,506)
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```
506           FORMAT(1X,"TRIED MORE THAN ESTIME TERMS IN SUM.",/)
C
C510             WRITE(6,520) I,J
520           FORMAT(1X,"I=",2X,I4,5X,"J=",2X,I4,/)
C
510            IF(J .GT. IMAX) IMAX=J
              IAVNTM=IAVNTM+J
               IF(ISKIP .NE  1) GO TO 530
C
             ARGCS=((K0*R)-(PI/4.0))/(200*PI)
              GO TO 7
530          ARGCS=(K0*R)-(PI/4.0)
C
C            WRITE(6,79) ARGCS
79          FORMAT(1X,"ARGCS= ",E15.8,/)
C
7            D1=COS(ARGCS)
C
C              WRITE(6,77)
77           FORMAT(1X,"GETTING HERE 7",/)
C
8            D2=-SIN(ARGCS)
C
C              WRITE(6,78)
78           FORMAT(1X,"GETTING HERE 8",/)
C
             D3=SQRT(2/(PI*K0*R))
             D5=D3*CMPLX(D1,D2)
            EZ0N=D5*SUMEZ
C
            SCRPEZ(I)=CABS(EZ0N)
C
             IF(SCRPEZ(I) .GT. TFLDMX) TFLDMX=SCRPEZ(I)
C
              ARG=K0*ABS(AA)
               CALL BESEL(0,ARG,BB,Y,BP,YP)
              J0=BB*BB
              Y0=Y*Y
              D1=3.E8**4.*8.854E-12**2.*8./(FREQ**2.*CURENT**2.*PI**3.)
9              ARG=K0*AA*COS(PHI)
11            D2=COS(ARG)
14            D3=SIN(ARG)
              D4=2.*PI*FREQ*CURENT/(4.*3.E8**2.*8.854E-12)
              D12=D4*CMPLX(D2,D3)
              D11=SUMEZ+D12
             XMAG=CABS(D11)*CABS(D11)
C
             ECHWPW(I)=D1*XMAG/(J0+Y0)
C
C              WRITE(6,1000) ECHWPW(I),PHI
C1000          FORMAT(1X,"ECHWPW=",2X,1E15.8,5X,"PHI=",2X,F7.3,/)
C
              IF(ECHWPW(I) .GT. SFLDMX) SFLDMX=ECHWPW(I)
C
700         CONTINUE
C
                  SCRPEZ(NTIMES+1)=SCRPEZ(1)
C
C             WRITE(6,1200)
1200         FORMAT(1X,////)
```

```
C
              WRITE(1,23)
23           FORMAT("PLOT THE SCATTERED FIELD?& Y/N")
              READ(1,12) IANS
               IF(IANS .EQ. 1H ) GO TO 505
               IF(IANS .NE. 1HY) GO TO 507
C
C * PLOT THE ECHO WIDTH PER WAVELNGTH VS ANGLE
C
505           CALL ECOPT(SFLDMX)
C
507           DO 900 I=1,NTIMES
                 PHI=(I-1.)*DGRENC
C               WRITE(6,800) ECHWPW(I),PHI
800            FORMAT(1X,"ECHWPW=",2X,1E15.8,5X,"PHI=",2X,F7.3,/)
900           CONTINUE
C
              WRITE(1,29)
29            FORMAT("PLOT THE TOTAL FIELD?& Y/N")
               READ(1,12) IANS
12            FORMAT(A1)
                IF(IANS .EQ. 1H ) GO TO 555
                IF(IANS .NE. 1HY) GO TO 950
C
555            WRITE(1,13)
13            FORMAT("NEW GRID?& Y/N")
               READ(1,12) IANSS
                IF(IANSS .EQ. 1H ) GO TO 560
                IF(IANSS .NE. 1HY) GO TO 565
C
560           CALL POLAR(0)
              CALL LABL(0)
C
565            WRITE(1,28)
28            FORMAT("ENTER PEN# FOR DATA.&")
               READ(1,*) IPND
               WRITE(25,22) IPND
22            FORMAT("SP",I1)
C
950           DO 600 I=1,NTIMES+1
                 PHI=((I-1)*DGRENC+180.)*PI/180.
                DBPRPT(I)=20.*ALOGT(SCRPEZ(I)/TFLDMX)
C
                  PHII=(I-1)*DGRENC
C                WRITE(6,33) DBPRPT(I),PHII
33            FORMAT(1X,"GAIN - POWER = ",E15.8," db",5X,"ANGLE = ",
     1                 F7.3,/)
C
                 IF(DBPRPT(I) .LT. -40.) DBPRPT(I)=-40.
                DUMMY=(DBPRPT(I)+40.)/40.
               IX=INT((XCR-DUMMY*COS(PHI)*RDS)*1000.)
               IY=INT((YCR-DUMMY*SIN(PHI)*RDS)*1000.)
                 IF(IANS .NE. 1H .AND. IANS .NE. 1HY) GO TO 600
                WRITE(25,24) IX,IY
24             FORMAT("PA",I5",",I5";PD")
600            CONTINUE
C
                WRITE(25,26)
26             FORMAT("PU")
C
```

```
      CALL LABEL(1)
C
      DO 940 I=1,NTIMES
         PHI=(I-1)*DGRENC
       WRITE(6,935) DBPRPT(I),PHI
935    FORMAT(1X,"POWER GAIN = ",E15.8," db",5X,"ANGLE =",
     1       F7.3," DEGREES",/)
940   CONTINUE
C
      WRITE(6,70)
C950   WRITE(6,945) SFLDMX
945   FORMAT(1X,"THE ECHWPW-MAX = ",E15.8,//)
C      WRITE(6,957) TFLDMX
957   FORMAT(1X,"THE TOTAL FIELD-MAX= ",E15.8)
C      WRITE(6,70)
C
      IAVNTM=IAVNTM/NTIMES
C     WRITE(6,947)
947   FORMAT(1X,"THE AVERAGE NUMBER OF TERMS IN THE INFINITE SUM")
C      WRITE(6,948) NTIMES
948   FORMAT(1X,"FOR ",I5," NUMBER OF ANGLES BETWEEN 0 & 360 IS",/)
C      WRITE(6,949) IAVNTM
949   FORMAT(1X,I5,"  TERMS.")
C      WRITE(6,70)
C      WRITE(6,951) IMAX
951   FORMAT(1X,"AND, THE MAX NUMBER OF TERMS WAS ",I5,"  TERMS.")
C
      CALL EXEC(23,5HSMP  ,4,ISLU)
C
      END
      END$
```

```
Y T=00004 IS ON CR00039 USING 00005 BLKS R=0000

FTN4.L
$EMA(CMBLK,0)
C *************************************
        SUBROUTINE CLSKY(N,M)
C *************************************
C
        COMPLEX A(6,7),F(6),
     1          SUM
C
C
        COMMON /CMBLK/ A,F,ECHWPW(3600),DGRENC,SCRPEZ(3600),
     1                 DBPRPT(3600)
C
        DATA PI/3.14159265/
C
C * CALCULATE FIRST ROW OF UPPER UNIT TRIANGULAR MATRIX
C
        DO 3 J=2,M
3       A(1,J)=A(1,J)/A(1,1)
C
C * CALCULATE OTHER ELEMENTS OF U AND L MATRICES
C
        DO 8 I=2,N
        J=I
        DO 5 II=J,N
        SUM=(0.0,0.0)
        JM1=J-1
        DO 4 K=1,JM1
4       SUM=SUM+A(II,K)*A(K,J)
5       A(II,J)=A(II,J)-SUM
        IP1=I+1
        DO 7 JJ=IP1,M
        SUM=(0.0,0.0)
        IM1=I-1
        DO 6 K=1,IM1
6       SUM=SUM+A(I,K)*A(K,JJ)
7       A(I,JJ)=(A(I,JJ)-SUM)/A(I,I)
8       CONTINUE
C
C * SOLVE FOR F(I) BY BACK SUBSTITUTION
C
        F(N)=A(N,N+1)
        L=N-1
        DO 10 NN=1,L
        SUM=(0.0,0.0)
        I=N-NN
        IP1=I+1
        DO 9 J=IP1,N
9       SUM=SUM+A(I,J)*F(J)
10      F(I)=A(I,M)-SUM
        RETURN
        END
        END$
```

```
I T=00004 IS ON CR00039 USING 00006 BLKS R=0000

 FTN4,L
 $EMA(CMBLK,0)
 C ****************************************
           SUBROUTINE ECOPT(XMAX)
 C ****************************************
 C
           COMPLEX A(6,7),F(6)
 C
 C
           COMMON /CMBLK/ A,F,ECHWPW(3600),DGRENC,SCRPEZ(3600),
      1                     DBPRPT(3600)
 C
 C
           DATA PI/3.14159265/
           NX0P=2.*1000
           NY0P=1.75*1000
 C
           NXSTP=0.8*1000
           NYSTP=0.5*1000
 C
           NXSTPS=9
           NYSTPS=10
 C
           IPEN=1
           WRITE(25,3) IPEN
 C
            NXEP=NX0P+NXSTPS*NXSTP
           WRITE(25,1) NX0P,NY0P
           WRITE(25,1) NXEP,NY0P
            WRITE(25,2)
 C
            NYEP=NY0P+NYSTPS*NYSTP
           WRITE(25,1) NX0P,NY0P
           WRITE(25,1) NX0P,NYEP
            WRITE(25,2)
 C
           DO 100 J=1,NXSTPS
             IX=NX0P+J*NXSTP
             IY1=NY0P-50
             IY2=NY0P+50
            WRITE(25,1) IX,IY1
            WRITE(25,1) IX,IY2
             WRITE(25,2)
 100       CONTINUE
 C
           DO 200 I=1,NYSTPS
             IY=NY0P+I*NYSTP
             IX1=NX0P-50
             IX2=NX0P+50
            WRITE(25,1) IX1,IY
            WRITE(25,1) IX2,IY
             WRITE(25,2)
 200      CONTINUE
 C
          CALL ECLBL
          CALL LABEL(1)
 C
          NPTS=IFIX(180./DGRENC)
```

```
C
        IPEN=3
        WRITE(25,3) IPEN
C
        DO 400 IPHI=1,NPTS
           IX=IFIX((IPHI-1.)*DGRENC/20.*NXSTP+2000.)
           IY=IFIX((ECHWPW(IPHI)/XMAX)/0.1*NYSTP+1750.)
         WRITE(25,1)IX,IY
400     CONTINUE
C
         WRITE(25,2)
1       FORMAT("PA",I5",",I5,";PD")
2       FORMAT("PU")
3       FORMAT("IN;SP",I1)
         END
         END$
```

```
:L T=00003 IS ON CR00039 USING 00015 BLKS R=0000

 FTN4.L
       SUBROUTINE BESEL(N,X,B,Y,BP,YP)
 C
 C     COMPUTE THE BESSEL FUNCTION OF ORDER N, N AN INTEGER N>OR=0
 C     WITH REAL ARGUMENT
 C     ALL EQUATION REFERENCES TO ABRAMOWITZ AND STEGUN
 C
       DIMENSION C0(7),C1(7),D0(7),D1(7),E0(7),E1(7),G0(7),G1(7)
       DATA C0/1.0,-2.2499997,1.2656208,-.3163866,.444479E-1,
      *-.39444E-2,.21E-3/
       DATA C1/.5,-.56249985,.21093573,-.3954289E-1,.443319E-2,
      *-.31761E-3,.1109E-4/
       DATA D0/.79788456,-.77E-6,-.55274E-2,-.9512E-4,.137237E-2,
      *-.72805E-03,.14476E-03/
       DATA D1/.79788456,.156E-5,.1659667E-1,.17105E-3,-.249511E-2,
      *.113653E-02,-.20033E-03/
       DATA E0/-.78539816,-.4166397E-1,-.3954E-4,.262573E-2,-.54125E-3,
      *-.29333E-03,.13558E-03/
       DATA E1/-2.35619449,.12499612,.565E-4,-.637879E-2,.74348E-3,
      *.79824E-03,-.29166E-03/
       DATA G0/.3674669,.6055937,-.7435038,.2530012,-.426121E-01,
      *.427916E-02,-.24846E-03/
       DATA G1/-.6366198,.2212091,2.168271,-1.316483,.312395,
      *-.400976E-01,.27873E-02/
       DATA PI/3.1415926/
       IFLG=0
       IF (N.LT.0) IFLG=1
       N=IABS(N)
       IF(ABS(X).LT.1.0E-10)GO TO 150
       IF(ABS(X).GT.3.0)GO TO 50
       X3SQ=X*X/9.0
       PROD=1.0
       B0=0.0
       B1=0.0
       CUM0=0.0
       CUM1=0.0
 C     SEE EQUATIONS 9.41 AND 9.44
       DO 5 I=1,7
       B0=B0+C0(I)*PROD
       B1=B1+C1(I)*PROD
       CUM0=CUM0+G0(I)*PROD
       CUM1=CUM1+G1(I)*PROD
       PROD=PROD*X3SQ
 5     CONTINUE
       B1=B1*X
       XC=2.0*SNGL(DLOG(DBLE(0.5*X)))/PI
       Y0=XC*B0+CUM0
       Y1=XC*B1+CUM1/X
       GO TO 100
 C     EQS 9.4.3 AND 9.4.6
 50    THROVX=3.0/X
       PROD=1.0
       F0=0.0
       F1=0.0
       THETA0=X
       THETA1=X
       DO 55 I=1,7
       F0=F0+D0(I)*PROD
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```
      F1=F1+D1(I)*PROD
      THETA0=THETA0+E0(I)*PROD
      THETA1=THETA1+E1(I)*PROD
      PROD=PROD*THROVX
55    CONTINUE
      SQRX=1.0/SNGL(DSQRT(DBLE(X)))
      B0=SQRX*F0*SNGL(DCOS(DBLE(THETA0)))
      B1=SQRX*F1*SNGL(DCOS(DBLE(THETA1)))
      Y0=SQRX*F0*SNGL(DSIN(DBLE(THETA0)))
      Y1=SQRX*F1*SNGL(DSIN(DBLE(THETA1)))
100   IF(N-1)101,105,110
101   B=B0
      BP=-B1
      Y=Y0
      YP=-Y1
      GO TO 200
105   B=B1
      BP=B0-B1/X
      Y=Y1
      YP=Y0-Y1/X
      GO TO 200
C     FOR RECURRSIVE DIRECTION COMMENTS SEE SECTION 9.12,P385
110   XN=N
      IF(XN.LT.ABS(X))GO TO 130
C     FOR X< N RECUR DOWNWARD
      BLAST=1.0
      BLASTP=0.0
      J=N+10
      DO 115 I=1,J
      XI=J-I
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST =BNEXT
      IF(I.NE.10)GO TO 115
      BLP=BLASTP
      Z=BNEXT
115   CONTINUE
      IF(ABS(B0).LT.ABS(B1))GO TO 117
      CORR=B0/BLASTP
      GO TO 118
117   CORR=-B1/BNEXT
118   B=BLP*CORR
      BNMIN1=Z*CORR
      GO TO 140
C     FOR X>N RECUR UPWARD
130   BLASTP=B0
      BLAST=B1
      DO 135 I=2,N
      XI=I-1
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST=BNEXT
135   CONTINUE
      B=BLAST
      BNMIN1=BLASTP
140   BP=BNMIN1-XN*B/X
      YLASTP=Y0
      YLAST=Y1
      DO 145 I=2,N
      XI=I-1
```

```
      YNEXT=2.0*XI*YLAST/X-YLASTP
      YLASTP=YLAST
      YLAST=YNEXT
145   CONTINUE
      Y=YLAST
      YP=YLASTP-XN*Y/X
      GO TO 200
150   B=0.0
      BP=0.0
      Y=0.0
      YP=0.0
      IF(N-1)155,160,200
155   B=1.0
      GO TO 200
160   BP=0.5
      B=0.5*X
200   IF (IFLG.EQ.0) RETURN
      B=(-1)**N*B
      N=-N
      RETURN
      END
      END*
```

```
P T=00004 IS ON CR00039 USING 00005 BLKS R=0000

C      I. MAIN Program
C
C         1) Reads inputs
C
C         2) Calls CELL
C            a) AN
C            b) NCELLS
C
C         3) Calls CLCRD
C            a) XN
C            b) YN
C
C         4) Calls ECHOW
C            a) W(PHI)/WAVELENGTH
C
C     II. ECHOW (need Ei and En)
C
C         1) Calls FDLTL
C            a) En
C
C         2) Calls FLDNC
C            a) Ei as a function of PHI
C
C    III. FLDTL (need Emi)
C
C         1) Calls FLDNC with PTOBS = 0
C            a) Emi
C
C         2) Sets up matrix and calls CLSKY
C            a) En
C
C >>> Have En and Ei >>> Have ECHWPW
C
C     IV. MAIN
C
C         5) Calls FLDCT
C            a) Es(rauo,phi) <<< Knowing En which is in an array in
C                EMA calculated in II.1
C
```

```
&M T=00004 IS ON CR00039 USING 00020 BLKS R=0000

FTN4,L
$EMA(BLKMM,0)
C ******************************
        PROGRAM RADMM
C ******************************
C
C                         =================
C                         ROBERT K. SCHNEIDER
C                         =================
C
C * This program uses the MOMENT METHOD as presented by JACK RICHMOND
C   ("Scattering by a Dielectric Cylinder of Arbitrary Cross Section
C   Shape", IEEE APS, Nov 1964, p. 334) to determine the Scattered
C   Field from and Echo Width of a dielectric cylindrical shell of
C   circular cross section.
C
C * The dimensions of the scatterer are read from a data file - DATAFL
C
      INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
      REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1     ECHWPW
C
      COMPLEX CMN,EINC1,EINC0,EINC2,
     1        ESCAT,
     2        TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3        TAU,VIN,VOUT,ONE,ZERO,TLFLD,A0,A1
C
      DIMENSION IBUFR(16)
C
      COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                EINC1(360),VIN(1851),ESCAT(360),
     2                ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3                EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
C
      EQUIVALENCE (NCELLS,NZ)
C
      DATA PI/3.14159265/
      DATA XCR/4.875/,YCR/3.875/,RDS/2.748/
      DATA IBUFR/2*0,2HDA,2HTA,2HFL,3*0,221B,7*0/
C
C * SET SPOOL FOR INPUT FROM IBUFR THROUGH ISLU
C
      CALL SPOPN(IBUFR,ISLU)
      CALL EXEC(22,1)
C
      READ(ISLU,*) AA,B,C,CURENT,R,FREQ,PERM,DGRENC,EPSLNR,NCELLS,
     1             HLFCEL
C
C * DETERMINE THE CELL STRUCTURE AND # OF CELLS
C
       W=0.0
      CALL CELL(W)
C****
         IF(HLFCEL .NE. 1) GO TO 50
C
       NCELLS=NCELLS/2
C****
```

```
C
C * DETERMINE THE COORDINATES OF THE CENTERS OF EACH CELL
C
C50       CALL CLCRD(W)
C
C * DETERMINE THE ECHO WIDTH PER WAVELENGTH
C
50        XMAX=0.
         CALL ECHOW(XMAX,W)
C
          NPTS=IFIX(360./DGRENC)
C
            WRITE(1,23)
23         FORMAT("PLOT THE SCATTERED FIELD?& Y/N")
            READ(1,12) IANS
12         FORMAT(A1)
             IF(IANS .EQ. 1H ) GO TO 210
             IF(IANS .NE. 1HY) GO TO 220
C
C * PLOT ECHO WIDTH PER WAVELENGHT
C
210      CALL ECOPT(XMAX)
C
C * DETERMINE THE FAR ZONE SCATTERED FIELD AT EACH DGRENC
C
220      CALL FLDCT
C
C * DETERMINE THE INCIDENT FIELD IN THE FAR ZONE
C
         CALL FLDNC(1,W)
C
         TFLDMX=0.0
C
         DO 260 I=1,NPTS
           TLFLD=EINC1(I)+ESCAT(I)
          SCRPEZ(I)=CABS(TLFLD)
           IF(SCRPEZ(I) .GT. TFLDMX) TFLDMX=SCRPEZ(I)
260      CONTINUE
C
           SCRPEZ(NPTS+1)=SCRPEZ(1)
C
          WRITE(1,24)
24       FORMAT("PLOT THE TOTAL FIELD?& Y/N")
         READ(1,12) IANS
          IF(IANS .EQ. 1H ) GO TO 230
          IF(IANS .NE. 1HY) GO TO 270
C
230       WRITE(1,25)
25       FORMAT("NEW GRID?& Y/N")
         READ(1,12) IANS
          IF(IANS .EQ. 1H ) GO TO 240
          IF(IANS .NE. 1HY) GO TO 250
C
240      CALL POLAR(0)
         CALL LABL(0)
C
250       WRITE(1,28)
28       FORMAT("ENTER PEN# FOR DATA.& ")
         READ(1,*) IPND
          WRITE(25,22) IPND
```

```
22        FORMAT("SP",I1)
C
          DO 700 I=1,NPTS+1
            PHI=((I-1)*DGRENC+180.)*PI/180.
           DBPRPT(I)=20.*ALOGT(SCRPEZ(I)/TFLDMX)
            IF(DBPRPT(I) .LT. -40.) DBPRPT(I)=-40.
            DUMMY=(DBPRPT(I)+40.)/40.
           IX=INT((XCR-DUMMY*COS(PHI)*RDS)*1000.)
           IY=INT((YCR-DUMMY*SIN(PHI)*RDS)*1000.)
            WRITE(25,29) IX,IY
29         FORMAT("PA",I5",","I5";PD")
700        CONTINUE
            WRITE(25,31)
31         FORMAT("PU")
C
270         WRITE(1,26)
26         FORMAT("HARD COPY OF CALCULATIONS?_ Y/N")
           READ(1,12) IANS
            IF(IANS .EQ. 1H ) GO TO 800
            IF(IANS .NE. 1HY) GO TO 500
C
800         WRITE(6,850) TFLDMX
850        FORMAT(1X,"TOTAL FIELD-MAX = ",E15.8,//)
C
           DO 870 I=1,NPTS
             PHI=(I-1)*DGRENC
             WRITE(6,860) PHI,ECHWPW(I),DBPRPT(I)
860         FORMAT(1X,"ANGLE= ",F5.2,5X,"ECHWPW= ",E15.8,5X,
     1                "DB GAIN= ",E15.8,/)
870        CONTINUE
C
            WRITE(6,27)
27         FORMAT(1X,///)
C
           DO 400 I=1,NPTS
             PHI=(I-1)*DGRENC
              XMAG=CABS((ESCAT(I)))
             WRITE(6,300) XMAG,PHI
300         FORMAT(1X,"MAGNITUDE ESCAT=",2X,1E15.8,5X,"PHI=",2X,F7.3,/)
400        CONTINUE
C
500       CALL EXEC(23,5HSMP  ,4,ISLU)
           END
           END$
```

```
  T=00004 IS ON CR00039 USING 00004 BLKS R=0000

FTN4,L
$EMA(BLKMN,0)
C ****************************************
      SUBROUTINE CELL(W)
C ****************************************
C
C * THIS SUBROUTINE DETERMINES THE # OF CELLS NEEDED AND THE
C     DIMENSION OF A CELL TO SPAN THE OBSTACLE.
C
C
      INTEGER NCELLS,NPTS,PTOBS,IPHI,NCELLS
C
      REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1      ECHWPW
C
      COMPLEX CMN,EINC1,EINC0,EINC2,
     1        ESCAT,
     2        TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3        TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
      DIMENSION IBUFR(16)
C
      COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                EINC1(360),VIN(1851),ESCAT(360),
     2                ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3                EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
C
      EQUIVALENCE (NCELLS,NZ)
C
C
      DATA PI/3.14159265/
C
C * THE # OF CELLS FOR THE CIRCULAR DIELECTRIC SHELL SCATTERER IS A
C     PARAMETER WHICH IS ASSUMED AT THE INITIATION OF THE PROGRAM AND
C     IS READ FROM THE DATA FILE
C
C * THE WIDTH OF EACH CELL MUST BE LESS THAN OR EQUAL TO
C     0.2 * WAVELENGTH / SQRT ( EPSLNR )
C
C * SINCE THE NUMBER OF CELLS IS A KNOWN VALUE AND THE SIZE OF THE
C     STRUCTURE IS DEFINED, THE WIDTH OF A 'SQUARE CELL', W, IS EASILY
C     DETERMINED TO BE
C
        W=2.*PI*C/NCELLS
C
C * RADIUS OF CIRCULAR CELL WITH EQUAL AREA AS 'SQUARE CELL', AN, IS
C
         ROUT=C
         RIN=C-W
       AN=SQRT((ROUT**2.-RIN**2.)/NCELLS)
C
C200    WRITE(6,100) W,AN,NCELLS
100    FORMAT(1X,"W=",2X,1E15.9,5X,"AN=",2X,1E15.9,5X,"NCELLS=",
     1        2X,I4,//)
C
      RETURN
       END
       END$
```

```
RD T=00004 IS ON CR00039 USING 00004 BLKS R=0000

  FTN4,L
  $EMA(BLKMM,0)
  C ***************************************
          SUBROUTINE CLCRD(W)
  C ***************************************
  C
  C * THIS SUBROUTINE CALCULATES THE COORDINATES OF THE CENTER OF
  C     EACH CELL.
  C
  C
          INTEGER NCELLS,NPTS,FTOBS,IPHI,NCELLS
  C
          REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
       1       ECHWPW
  C
          COMPLEX CMN,EINC1,EINC0,EINC2,
       1         ESCAT,
       2         TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
       3         TAU,VIN,VOUT,ONE,ZERO,A0,A1
  C
          DIMENSION IBUFR(16)
  C
          COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
       1                 EINC1(360),VIN(3702),ESCAT(360),XNN(3702),
       2                 YNN(3702),ECHWPW(360),K0,K2,VOUT(3702),TAU(3702),
       3                 EINC2,SCRPEZ(361),DBPRPT(361),A0(3702),A1(3702)
  C
          EQUIVALENCE (NCELLS,NZ)
  C
  C
  C
          DATA PI/3.14159265/
  C
  C * RADIUS OUT TO CENTER OF CELL n
  C
          RN=C-(W/2.)
  C
  C * INCREMENT ANGLE FOR CELL LOCATIONS
  C
          THETAN=2.*PI/NCELLS
  C
  C        WRITE(6,50) THETAN
  50      FORMAT(1X,"THETAN=",2X,1E15.9,//)
  C
  C * DETERMINE THE CORDINATES OF THE CENTER OF EACH CELL n
  C
          DO 10 N=1,NCELLS
            THETA=(N-1)*THETAN
           XNN(N)=RN*COS(THETA)
           YNN(N)=RN*SIN(THETA)
  C
  C         WRITE(6,100) XNN(N),YNN(N),N
  100      FORMAT(1X,"XNN=",2X,1E15.9,5X,"YNN=",2X,1E15.9,5X,"N=",2X,I4,/)
  10      CONTINUE
  C
          RETURN
           END
           END$
```

```
W T=00004 IS ON CR00039 USING 00008 BLKS R=0000

 FTN4,L
 $EMA(BLKMN,0)
 C ****************************************
         SUBROUTINE ECHOW(XMAX,W)
 C ****************************************
 C
 C * THIS SUBROUTINE CALCULATES THE ECHO WIDTH PER UNIT WAVELENGHT,
 C     RELATIVE TO THE INCIDENT FIELD AT THE CENTER OF THE OBSTACLE,
 C     FROM A DIELECTRIC CYLINDRICAL SHELL OF CIRCULAR CROSS SECTION
 C     IN THE PRESENCE OF A RADIATING CURRNT FILAMENT.
 C
 C
 C
         INTEGER NCELLS,NPTS,PTOBS,IPHI,NCELLS
 C
         REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
      1       ECHWPW
 C
         COMPLEX CNN,EINC1,EINC0,EINC2,
      1          ESCAT,
      2          TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
      3          TAU,VIN,VOUT,ONE,ZERO,DUMMY4,A0,A1
 C
         DIMENSION IBUFR(16)
 C
         COMMON /BLKNM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
      1                 EINC1(360),VIN(1851),ESCAT(360),
      2                 ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
      3                 EINC2,SCRPE2(361),DBPRPT(361),A0(1851),A1(1851)
 C
         EQUIVALENCE (NCELLS,N2)
 C
 C
         DATA PI/3.14159265/
 C
 C * CALL FLDTL TO DETERMINE THE TOTAL FIELD IN THE OBSTACLE
 C     DUE TO THE INCIDENT FIELD UPON THE OBSTACLE
 C
         CALL FLDTL(W)
 C
 C * CALL FLDNC TO DETERMINE THE INCIDENT FIELD AT THE CENTER OF
 C     THE OBSTACLE.
 C
         CALL FLDNC(2,W)
 C
         NPTS=IFIX(360./DGRENC)
 C
 C        DO 7 I=1,NPTS
 C          WRITE(6,5) EINC2
 5         FORMAT(1X,"ECHOW : EINC2=",2X,1E15.8,2X,E15.8,/)
 C7         CONTINUE
 C
 C * DETERMINE THE MAGNITUDE SQUARED OF THE INCIDENT FIELD
 C
          EMAGI=CABS((EINC2))*CABS((EINC2))
 C
         DUMMY1=K0*PI**2.*FREQ/(3.E8*EMAGI)
 C
```

```
C * LOOP NCELLS TIMES FOR THE "SCATTERED FIELD" IN THE FAR ZONE.
C
      DO 20 IPHI=1,NPTS
         PHI=(IPHI-1.)*DGRENC*PI/180.
C
       DUMMY4=(0.,0.)
C
      RN=C-(W/2.)
      THETAN=2.*PI/NCELLS
C
      DO 10 N=1,NCELLS
         THETA=(N-1)*THETAN
          XNN=RN*COS(THETA)
          YNN=RN*SIN(THETA)
        ARGJ1=K0*AN
         CALL BESEL(1,ARGJ1,BB,Y,BP,YP)
        J1=BB
        ARGH=K0*(XNN*COS(PHI)+YNN*SIN(PHI))
        DUMMY2=COS(ARGH)
        DUMMY3=SIN(ARGH)
       DUMMY4=DUMMY4+(EPSLNR-1.)*VOUT(N)*AN*J1*CMPLX(DUMMY2,DUMMY3)
10    CONTINUE
C
       EMAGN=CABS(DUMMY4)*CABS(DUMMY4)
C
      ECHWPW(IPHI)=DUMMY1*EMAGN
CC
       IF(ECHWPW(IPHI) .GT. XMAX) XMAX=ECHWPW(IPHI)
CC
C
          PHII=PHI*180./PI
C
C      WRITE(6,100) ECHWPW(IPHI),PHII
100   FORMAT(1X,"ECHWPW=",2X,1E15.9,5X,"PHI=",2X,F7.3,/)
C
20    CONTINUE
C
      RETURN
       END
       END$
```

L T=00004 IS ON CR00039 USING 00003 BLKS R=0000

```
FTN4,L
$EMA(BLKMN,0)
C ********************************
      SUBROUTINE FLDTL(W)
C ********************************
C
C * THIS SUBROUTINE CALCULATES THE TOTAL FIELD IN THE OBSTACLE
C
C
      INTEGER NCELLS,NPTS,PTOBS.IPHI,NCELLS
C
      REAL XNN,YNN,EMAGI,J1,J0,K0,K2:Y0,Y1,EMAGN,
     1     ECHWPW
C
      COMPLEX CMN,EINC1,EINC0,EINC2,
     1        ESCAT,
     2        TAU1,ALMDA.ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3        TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
      DIMENSION IBUFR(16)
C
      COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGFENC,EPSLNR,AN,NCELLS,
     1                EINC1(360),VIN(1851).ESCAT(360),
     2                ECHWPW(360),K0,K2,VOUT(1251),TAU(1251),
     3                EINC2,SCRPEZ(361),DBPRPT(361),A0(1251),A1(1251)
C
      EQUIVALENCE (NCELLS,NZ)
C
C
C
      DATA PI/3.14159265/
C
C * OBTAIN THE FIELD INCIDENT UPON THE OBSTACLE
C
      CALL FLDNC(0,W)
C
C      DO 5 I=1,NCELLS
C        WRITE(6,7) EINC0(I),I
7      .FORMAT(1X,"FLDTL : EINC0=",2X,1E15.8,2X,E15.8,5X,"I=",2X,
     1        I4,/)
C5      CONTINUE
C
       RN=C-(W/2.)
       THETAN=2.*PI/NCELLS
C
C     DO 30 M=1,NCELLS
CC
            M=1
              THETAN=(M-1)*THETAN
             XNM=RN*COS(THETAM)
             YNM=RN*SIN(THETAM)
CC
C
      DO 20 N=1,NCELLS
C
C * DETERMINE THE WEIGHT Cmn ON En
C
          THETA=(N-1)*THETAN
```

AD-A115 517 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 17/9
THE EFFECT OF RADOME SCATTERING ON ECM ANTENNA PATTERNS.(U)
DEC 81 R K SCHNEIDER
UNCLASSIFIED AFIT/GE/EE/81D-52 NL

2 of 2
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A115517

END
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DTIC

```
            XNN=RN*COS(THETA)
            YNN=RN*SIN(THETA)
           IF(N .NE. 1) GO TO 10
C
            DUMMY1=K0*AN
             CALL BESEL(1,DUMMY1,BB,Y,BP,YP)
            J1=BB
            Y1=Y
C
C            WRITE(6,8) J1,Y1,BB
8           FORMAT(1X,"FLDTL : J1=",2X,1E15.8,5X,"Y1=",2X,1E15.8,5X,
     1              "BB=",2X,1E15.8,/)
C
           CMN=1.+(EPSLNR-1.)*((PI*K0*AN/2.)*(CMPLX(Y1,J1))+1.)
C          A(M,N)=CMN
              IF(M .NE. 1) GO TO 20
           TAU(N)=CMN
            GO TO 20
C
10          DUMMY1=PI*K0*AN/2.
            RMN=SQRT((XNM-XNN)*(XNM-XNN) +
     1                (YNM-YNN)*(YNM-YNN))
            ARGH=K0*RMN
            ARGJ=K0*AN
             CALL BESEL(1,ARGJ,BB,Y,BP,YP)
            J1=BB
             CALL BESEL(0,ARGH,BB,Y,BP,YP)
            J0=BB
            Y0=Y
C
           CMN=DUMMY1*(EPSLNR-1.)*J1*CMPLX(Y0,J0)
C            IF(N .NE. 1) GO TO 20
           TAU(N)=CMN
C
C          A(M,N)=CMN
20        CONTINUE
C          A(M,N+1)=-EINC0(M)
C30        CONTINUE
C
C * NOW THAT THE MATRIX HAS BEEN FORMED, SOLVE FOR En
C
          MCELLS=NCELLS+1
C           WRITE(6,250) MCELLS,NCELLS
250      FORMAT(1X,"MCELLS=",2X,I4,5X,"NCELLS=",2X,I4,//)
C         DO 400 M=1,NCELLS
C
C         DO 300 N=1,MCELLS
C           WRITE(6,350) A(M,N),M,N
350      FORMAT(1X,"FLDTL : A=",2X,1E15.9,2X,E15.9,5X,"M=",2X,I4,
     1          5X,"N=",2X,I4,/)
C300       CONTINUE
C           WRITE(6,375)
375        FORMAT(1X,///)
C400       CONTINUE
C
C         DO 600 I=1,NCELLS
C           WRITE(6,700) TAU(I),I
700        FORMAT(1X,"TAU=",2X,1E15.9,2X,E15.9,5X,I4,/)
C600       CONTINUE
C
```

```
        CALL TPLZ(1,XNORM,IER)
C
         WRITE(1,500) IER
500     FORMAT(1X,"IER=",2X,I4)
C
C * En ARE CONTAINED IN THE ARRAY VOUT OF DIMENSION NCELLS
C
C            DO 100 I=1,NCELLS
C             WRITE(6,200) VOUT(I),I
200          FORMAT(1X,"FLDTL : VOUT==",2X,1E15.9,2X,E15.9,5X,"I=",
     1              2X,I4,/)
C100          CONTINUE
C
        RETURN
         END
         END#
```

```
: T=00004 IS ON CR00039 USING 00009 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
C ******************************
            SUBROUTINE FLDNC(PTOBS,W)
C ******************************
C
C * THIS SUBROUTINE CALCULATES THE INCIDENT FIELD ON THE OBSTACLE,
C    AT THE FAR.ZONE POINT, AND/OR AT THE CENTER OF THE OBSTACLE.
C
C
          INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
          REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1         ECHWPW
C
          COMPLEX CNN,EINC1,EINC0,EINC2,
     1             ESCAT,
     2             TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3             TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
          DIMENSION IBUFR(16)
C
          COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                    EINC1(360),VIN(1851),ESCAT(360),
     2                    ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3                    EINC2,SCRPE2(361),DBPRPT(361),A0(1851),A1(1851)
C
          EQUIVALENCE (NCELLS,NZ)
C
C
C
          DATA PI/3.14159265/
C
          CURENT=1.0
C
          NPTS=IFIX(360./DGRENC)
C
C * WAVE # IN FREE SPACE AND IN THE OBSTACLE
C
          K0=2.*PI*FREQ/3.E8
          K2=K0*SQRT(EPSLNR)
C
C * EPSLN0 = 8.854E-12
C
           RN=C-(W/2.)
           THETAN=2.*PI/NCELLS
C
          DUMMY1=-(K0**2./(4.*2.*PI*FREQ*8.854E-12))*CURENT
C
C * IF OBSERVATION POINT AT PARTICULAR CELL, PTOBS = 0.  IF OBSER-
C    VATION POINT IN FAR ZONE AT SOME ANGLE PHI, PTOBS = 1.  IF
C    OBSERVATION POINT AT CENTER OF OBSTACLE, PTOBS = 2.
C
          IF(PTOBS .EQ. 1) GO TO 20
C
          IF(PTOBS .EQ. 2) GO TO 50
C
C * DETERMINE THE INCIDENT FIELD ON THE OBSTACLE AT EACH CELL
```

```
C     LOCATION, (Xn,Yn)
C
C * LOOP NCELLS TIMES FOR INCIDENT FIELD
C
        DO 10 I=1,NCELLS
             THETA=(I-1)*THETAN
            XNN=RN*COS(THETA)
            YNN=RN*SIN(THETA)
           DUMMY0=(YNN-0.)*(YNN-0.)
          ARGH=K0*SQRT((XNN-AA)*(XNN-AA)+DUMMY0)
           CALL BESEL(0,ARGH,BB,Y,BP,YP)
          J0=BB
          Y0=Y
C
C         WRITE(6,5) J0,Y0,BB
5        FORMAT(1X,"FLDNC : J0=",2X,1E15.8,5X,"Y0=",2X,1E15.8,5X,
     1              "BB=",2X,1E15.8,/)
C
         EINC0=DUMMY1*CMPLX(J0,-Y0)
         VIN(I)=EINC0
C
C         WRITE(6,100) EINC0,I,VIN(I)
100      FORMAT(1X,"EINC0=",2X,1E15.9,2X,E15.9,5X,"I=",I4,
     1             5X,"VIN=",2X,1E15.9,2X,E15.9,/)
10      CONTINUE
         GO TO 40
C
C * DETERMINE THE INCIDENT FIELD AT THE FAR ZONE POINT DUE TO
C     THE CURRENT FILAMENT AT RAU PRIME.  USE LARGE ARGUEMENT
C     ASYMPTOTIC EXPANSION FOR THE HANKEL FUNCTION.
C
20      DO 30 J=1,NPTS
          PHI=(J-1)*DGRENC*PI/180.
           ARGH=K0*(R-AA*COS(PHI))-PI/4.
          D1=COS(ARGH)
          D2=-SIN(ARGH)
          D3=SQRT(2./(K0*PI*R))
C
         EINC1(J)=DUMMY1*D3*CMPLX(D1,D2)
C         WRITE(6,200) EINC1(J),J
200      FORMAT(1X,"FLDNC : EINC1=",2X,1E15.8,2X,E15.8,5X,"I=",2X,
     1             I4,/)
30      CONTINUE
C
         GO TO 40
C
C * DETERMINE THE FIELD INCIDENT AT THE CENTER OF THE OBSTACLE.
C
50        ARGH=K0*ABS(AA)
           CALL BESEL(0,ARGH,BB,Y,BP,YP)
          J0=BB
          Y0=Y
C
         EINC2=DUMMY1*CMPLX(J0,-Y0)
C
40      RETURN
         END
         END$
```

```
T T=00004 IS ON CR00039 USING 00010 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
 C ****************************************
          SUBROUTINE ECOPT(XMAX)
 C ****************************************
 C
 C * THIS SUBROUTINE PLOTS THE NORMALIZED ECHO WIDTH PER WAVELENGTH
 C     VS. ANGLE PHI ON A LINEAR PLOT.
 C
 C
          INTEGER NCELLS,NPTS,PTODS,IPHI,MCELLS
 C
          REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
      1        ECHWPW
 C
          COMPLEX CMN,EINC1,EINC0,EINC2,
      1           ESCAT,
      2           TAU1,ALNDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
      3           TAU,VIN,VOUT,ONE,ZERO,A0,A1
 C
          DIMENSION IBUFR(16)
 C
          COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
      1                  EINC1(360),VIN(1851),ESCAT(360),
      2                  ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
      3                  EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
 C
          EQUIVALENCE (NCELLS,NZ)
 C
 C
          DATA PI/3.14159265/
          NX0P=2.*1000
          NY0P=1.75*1000
 C
          NXSTP=0.8*1000
          NYSTP=0.5*1000
 C
          NXSTPS=9
          NYSTPS=10
 C
          IPEN=1
          WRITE(25,3) IPEN
 C
           NXEP=NX0P+NXSTPS*NXSTP
          WRITE(25,1) NX0P,NY0P
          WRITE(25,1) NXEP,NY0P
           WRITE(25,2)
 C
           NYEP=NY0P+NYSTPS*NYSTP
          WRITE(25,1) NX0P,NY0P
          WRITE(25,1) NX0P,NYEP
           WRITE(25,2)
 C
          DO 100 J=1,NXSTPS
            IX=NX0P+J*NXSTP
            IY1=NY0P-50
            IY2=NY0P+50
           WRITE(25,1) IX,IY1
```

```
       WRITE(25,1) IX,IY2
        WRITE(25,2)
100     CONTINUE
C
        DO 200 I=1,NYSTPS
          IY=NY0P+I*NYSTP
          IX1=NX0P-50
          IX2=NX0P+50
         WRITE(25,1) IX1,IY
         WRITE(25,1) IX2,IY
          WRITE(25,2)
200    CONTINUE
C
       CALL ECLBL
       CALL LABEL(1)
C
       NPTS=IFIX(180./DGRENC)
C
       IPEN=3
       WRITE(25,3) IPEN
C
       DO 400 IPHI=1,NPTS
          IX=IFIX((IPHI-1.)*DGRENC/20.*NXSTP+2000.)
          IY=IFIX((ECHWPW(IPHI)/XMAX)/0.1*NYSTP+1750.)
        WRITE(25,1)IX,IY
400    CONTINUE
C
        WRITE(25,2)
1      FORMAT("PA",I5",",I5,";PD")
2      FORMAT("PU")
3      FORMAT("IN;SP",I1)
        END
        END$
```

```
T T=00004 IS ON CR00039 USING 00004 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
 C *******************************
         SUBROUTINE FLDCT
 C *******************************
 C
 C * THIS SUBROUTINE USES THE MOMENT METHOD TO DETERMINE THE SCATTERED
 C     FIELD FROM SOME DEFINED OBSTACLE
 C
 C
         INTEGER NCELLS,NPTS,PTOBS,IPHI,NCELLS
 C
         REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
      1       ECHWPW
 C
         COMPLEX CMN,EINC1,EINC0,EINC2,
      1         ESCAT,
      2         TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
      3         TAU,VIN,VOUT,ONE,ZERO,DUMMY8,A0,A1
 C
         DIMENSION IBUFR(16)
 C
         COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
      1                  EINC1(360),VIN(1851),ESCAT(360),
      2                  ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
      3                  EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
 C
         EQUIVALENCE (NCELLS,NZ)
 C
 C
 C
         DATA PI/3.14159265/
 C
 C * THE LARGE ARGUENENT ASYMPTOTIC EXPANSION FOR THE HANKEL FUNCTION
 C     IS USED. SEE RICHMOND -----
 C
 C * LOOP NPTS TIMES TO OBTAIN THE SCATTERED FIELD AT EACH DGRENC
 C
         NPTS=IFIX(360./DGRENC)
 C
           RN=C-(W/2.)
           THETAN=2.*PI/NCELLS
 C
         DO 20 IPHI=1,NPTS
           PHI=(IPHI-1)*DGRENC*PI/180.
           DUMMY1=(K0*R)-(PI/4.)
           DUMMY2=-COS(DUMMY1)
           DUMMY3=-SIN(DUMMY1)
           DUMMY4=SQRT(PI*K0*0.5/R)
 C
 C * LOOP NCELLS TIMES
 C
          DUMMY8=(0.,0.)
 C
         DO 10 N=1,NCELLS
            THETA=(N-1)*THETAN
             XNN=RN*COS(THETA)
             YNN=RN*SIN(THETA)
```

```
        DUMMY5=K0*(XNN*COS(PHI)+YNN*SIN(PHI))
        DUMMY6=COS(DUMMY5)
        DUMMY7=SIN(DUMMY5)
        ARGJ1=K0*AN
        CALL  BESEL(1,ARGJ1,BB,Y,BP,YP)
       J1=BB
       DUMMY8=DUMMY8+(EPSLNR-1.)*VOUT(N)*AN*J1*CMPLX(DUMMY6,DUMMY7)
10    CONTINUE
C
       ESCAT(IPHI)=DUMMY4*CMPLX(DUMMY3,DUMMY2)*DUMMY8
C
20    CONTINUE
C
      RETURN
       END
       END$
```

```
 T=00004 IS ON CR00039 USING 00006 BLKS R=0000

FTN4,L
$EMA(BLKMM,0)
C ***********************************************************
        SUBROUTINE TPLZ(MM,XNORM,IER)
C ***********************************************************
C
C ===========================================================
C
C * From
C
C    "Antenna Theory and Design"
C      Warren L. Stutzman and Gary A. Thiele
C       John Wiley & Sons, New York, 1981
C       Appendix G.7 pp. 579-581
C
C ===========================================================
C
C * PURPOSE
C       TO SOLVE A SYSTEM INVOLVING A TOEPLITZ MATRIX.  TPLZ REQUIRES
C       ONLY 5N STORAGE LOCATIONS FOR AN N BY N MATRIX.
C
C * REMARKS
C
C A toeplitz matrix has the first row equal to the first column.
C  All elements along the main diagonal are equal.  Any diagonal
C  off the main diagonal will have this same property.
C
C * DESCRIPTION OF PARAMETERS
C
C   NZ      -ORDER OF MATRIX
C   TAU     -FIRST ROW OR COLUMN OF THE TOEPLITZ MATRIX (VECTOR
C             LENGTH NZ)
C   A0,A1   -VECTORS OF LENGTH NZ NEEDED FOR SCRATCH AREA
C   VIN     -FOR THE MATRIX EQUATION (Z)(I)=(V), VIN IS V.  (Z,I,
C             AND V MAY BE THOUGHT OF AS GENERALIZED IMPEDANCES,
C             CURRENTS, AND VOLTAGES, RESPECTIVELY). V IS A NZ BY
C             MM MATRIX.
C   MM      -NUMBER OF COLUMN VECTORS ON THE RIGHT SIDE OF MATRIX
C             EQUATION (Z)(I)=(V) (USUALLY 1).
C   XNORM   -UPON RETURN THIS IS INFINITE MATRIX NORM OF INVERSE.
C   IER     -ERROR CODE WHICH IS 0 IF NO TROUBLE.
C
C
        INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
        REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1       ECHWPW
C
        COMPLEX CMN,EINC1,EINC0,EINC2,
     1          ESCAT,
     2          TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3          TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
        DIMENSION IBUFR(16)
C
        COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                EINC1(360),VIN(1851),ESCAT(360),
     2                ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
```

```
     3                    EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
C
      EQUIVALENCE (NCELLS,NZ)
C
C
      DATA ONE/(1E0,0E0)/, ZERO/(0E0,0E0)/
      DATA ONNE/1E0/, ZRRO/0.E0/
C
C      WRITE(6,90) NZ,MM
90    FORMAT(1X,"NZ=",2X,I4,5X,"MM=",2X,I4,/)
       DO 150 I=1,NZ
C        WRITE(6,100) TAU(I),VIN(I),I
100   FORMAT(1X,"TPLZ : TAU=",2X,1E15.8,2X,E15.8,5X,"VIN=",2X,1E15.8,
     1        2X,E15.8,5X,"I=",I4,/)
150    CONTINUE
      N=NZ-1
      IER=0
C
C * NORMALIZE INPUT MATRIX
C
       TAU1=TAU(1)
      DO 2000 II=1,N
2000   TAU(II)=TAU(II+1)/TAU1
C
C * THE FOLLOWING CALCULATES THE ITERATIVE VARIABLES TO OBTAIN
C     A0(N) AND ALMDA
C
C * NOTE :  VECTOR A0(I) HAS I ELEMENTS AND IS STORED AS A0(I,J),
C             J=1,N
C
       ALMDA=ONE-TAU(1)*TAU(1)
       A0(1)=-TAU(1)
       I=2
    1  KK=I-1
       ALPHA=ZERO
      DO 2 M=1,KK
       LL=I-M
    2 ALPHA=ALPHA+A0(M)*TAU(LL)
       ALPHA=-(ALPHA+TAU(I))
        IF(CABS(ALPHA) .EQ. 0.D0) GO TO 15
       COEF=ALPHA/ALMDA
       ALMDA=ALMDA-COEF*ALPHA
      DO 3 J=1,KK
       L=I-J
    3 A1(J)=A0(J)+COEF*A0(L)
      DO 7 J=1,KK
    7 A0(J)=A1(J)
       A0(I)=COEF
        IF(I .GE. N) GO TO 5
    4 I=I+1
       GO TO 1
C
C * THE FOLLOWING COMPUTES VALUES OF EACH ELEMENT OF THE INVERSE
C
    5 NH=(NZ+1)/2
       FAC=ALMDA*TAU1
       XNORM=ZRRO
       NP=NZ+1
      DO 51 I=1,NH
       XNM=ZRRO
```

```
      IF(I .NE. 1) GO TO 52
      A1(1)=ONE/FAC
      XNM=CABS(A1(1))
     DO 53 J=2,NZ
      A1(J)=A0(J-1)/FAC
  53 XNM=CABS(A1(J))+XNM
      GO TO 54
  52 XNM=ZERO
      JH=I-1
      C1=A0(JH)
      NNPI=NP-I
      C2=A0(NNPI)
     DO 55 JJ=1,N
       J=NP-JJ
       INPJ=NP-J
       JL=J-1
       A1(J)=A1(JL)+(C1*A0(JL)-C2*A0(INPJ))/FAC
  55 XNM=CABS(A1(J))+XNM
      A1(1)=A0(I-1)/FAC
      XNM=XNM+CABS(A1(1))
  54   IF(XNM .GT. XNORM) XNORM=XNM
C
C * MATRIX MULTIPLY
C
     DO 56 II=1,MM
       ID=(II-1)*NZ
       V=ZERO
       V1=ZERO
     DO 57 J=1,NZ
       NIDJ=ID+J
       V2=VIN(NIDJ)
       V=V+V2*A1(J)
       KNPJ=NP-J
  57 V1=V1+V2*A1(KNPJ)
      NIDI=ID+I
      VOUT(NIDI)=V
C      WRITE(6,225) VOUT(NIDI),NIDI
225   FORMAT(1X,"TPLZ : VOUT=",2X,1E15.8,2X,E15.8,5X,"NI=",2X,I4,/)
      KIDNPI=ID+NP-I
  56 VOUT(KIDNPI)=V1
C      WRITE(6,250) VOUT(KIDNPI),KIDNPI,VOUT(NIDI),NIDI
250   FORMAT(1X,"TPLZ : VOUT=",2X,1E15.8,2X,E15.8,5X,"KI=",2X,I4,
     1        3X,"VOUT=",2X,1E15.8,2X,E15.8,5X,"NI=",2X,I4,/)
C      WRITE(6,251) VOUT(KIDNPI),KIDNPI
251   FORMAT(1X,"TPLZ : VOUT=",2X,1E15.8,2X,E15.8,5X,"KI=",2X,I4,/)
C
51    CONTINUE
C
      DO 650 I=1,NCELLS
C      WRITE(6,600) VOUT(I),I
600   FORMAT(1X,"TPLZ :: VOUT=",2X,1E15.8,2X,E15.8,5X,"I=",2X,I4,/)
650   CONTINUE
C
      RETURN
C
 15    WRITE(6,700)
 700  FORMAT(1X,"ERROR HAS OCCURRED. MATRIX IS STRONGLY NONSINGULAR")
       IER=I
C
      RETURN
```

END
END\#

```
:L T=00003 IS ON CR00039 USING 00015 BLKS R=0000

FTN4,L
      SUBROUTINE BESEL(N,X,B,Y,BP,YP)
C
C     COMPUTE THE BESSEL FUNCTION OF ORDER N. N AN INTEGER N>OR=0
C     WITH REAL ARGUMENT
C     ALL EQUATION REFERENCES TO ABRAMOWITZ AND STEGUN
C
      DIMENSION C0(7),C1(7),D0(7),D1(7),E0(7),E1(7),G0(7),G1(7)
      DATA C0/1.0,-2.2499997,1.2656208,-.3163866,.444479E-1,
     *-.39444E-2,.21E-3/
      DATA C1/.5,-.56249985,.21093573,-.3954289E-1,.443319E-2,
     *-.31761E-3,.1109E-4/
      DATA D0/.79788456,-.77E-6,-.55274E-2,-.9512E-4,.137237E-2,
     *-.72805E-03,.14476E-03/
      DATA D1/.79788456,.156E-5,.1659667E-1,.17105E-3,-.249511E-2,
     *.113653E-02,-.20033E-03/
      DATA E0/-.78539816,-.4166397E-1,-.3954E-4,.262573E-2,-.54125E-3,
     *-.29333E-03,.13558E-03/
      DATA E1/-2.35619449,.12499612,.565E-4,-.637879E-2,.74348E-3,
     *.79824E-03,-.29166E-03/
      DATA G0/.3674669,.6055937,-.7435038,.2530012,-.426121E-01,
     *.427916E-02,-.24846E-03/
      DATA G1/-.6366198,.2212091,2.168271,-1.316483,.312395,
     *-.400976E-01,.27873E-02/
      DATA PI/3.1415926/
      IFLG=0
      IF (N.LT.0) IFLG=1
      N=IABS(N)
      IF(ABS(X).LT.1.0E-10)GO TO 150
      IF(ABS(X).GT.3.0)GO TO 50
      X3SQ=X*X/9.0
      PROD=1.0
      B0=0.0
      B1=0.0
      CUM0=0.0
      CUM1=0.0
C     SEE EQUATIONS 9.41 AND 9.44
      DO 5 I=1,7
      B0=B0+C0(I)*PROD
      B1=B1+C1(I)*PROD
      CUM0=CUM0+G0(I)*PROD
      CUM1=CUM1+G1(I)*PROD
      PROD=PROD*X3SQ
5     CONTINUE
      B1=B1*X
      XC=2.0*SNGL(DLOG(DBLE(0.5*X)))/PI
      Y0=XC*B0+CUM0
      Y1=XC*B1+CUM1/X
      GO TO 100
C     EQS 9.4.3 AND 9.4.6
50    THROVX=3.0/X
      PROD=1.0
      F0=0.0
      F1=0.0
      THETA0=X
      THETA1=X
      DO 55 I=1,7
      F0=F0+D0(I)*PROD
```

```
      F1=F1+D1(I)*PROD
      THETA0=THETA0+E0(I)*PROD
      THETA1=THETA1+E1(I)*PROD
      PROD=PROD*THROVX
55    CONTINUE
      SQRX=1.0/SNGL(DSQRT(DBLE(X)))
      B0=SQRX*F0*SNGL(DCOS(DBLE(THETA0)))
      B1=SQRX*F1*SNGL(DCOS(DBLE(THETA1)))
      Y0=SQRX*F0*SNGL(DSIN(DBLE(THETA0)))
      Y1=SQRX*F1*SNGL(DSIN(DBLE(THETA1)))
100   IF(N-1)101,105,110
101   B=B0
      BP=-B1
      Y=Y0
      YP=-Y1
      GO TO 200
105   B=B1
      BP=B0-B1/X
      Y=Y1
      YP=Y0-Y1/X
      GO TO 200
C     FOR RECURRSIVE DIRECTION COMMENTS SEE SECTION 9.12,P385
110   XN=N
      IF(XN.LT.ABS(X))GO TO 130
C     FOR X< N RECUR DOWNWARD
      BLAST=1.0
      BLASTP=0.0
      J=N+10
      DO 115 I=1,J
      XI=J-I
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST =BNEXT
      IF(I.NE.10)GO TO 115
      BLP=BLASTP
      Z=BNEXT
115   CONTINUE
      IF(ABS(B0).LT.ABS(B1))GO TO 117
      CORR=B0/BLASTP
      GO TO 118
117   CORR=-B1/BNEXT
118   B=BLP*CORR
      BNMIN1=Z*CORR
      GO TO 140
C     FOR X>N RECUR UPWARD
130   BLASTP=B0
      BLAST=B1
      DO 135 I=2,N
      XI=I-1
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST=BNEXT
135   CONTINUE
      B=BLAST
      BNMIN1=BLASTP
140   BP=BNMIN1-XN*B/X
      YLASTP=Y0
      YLAST=Y1
      DO 145 I=2,N
      XI=I-1
```

```
      YNEXT=2.0*XI*YLAST/X-YLASTP
      YLASTP=YLAST
      YLAST=YNEXT
145   CONTINUE
      Y=YLAST
      YP=YLASTP-XN*Y/X
      GO TO 200
150   B=0.0
      BP=0.0
      Y=0.0
      YP=0.0
      IF(N-1)155,160,200
155   B=1.0
      GO TO 200
160   BP=0.5
      B=0.5*X
200   IF (IFLG.EQ.0) RETURN
      B=(-1)**N*B
      N=-N
      RETURN
      END
      END$
```

Appendix G. Computer Program for Moment Method

This appendix contains the computer program which implements the moment method solution. This program is a collection of routines written in FORTRAN 4. Each subroutine is self-explanatory; the road map, RDMAP, gives some indication of how the final answers are arrived at. The computer system used was the Hewlett-Packard 27MX series microcomputer.

```
:: T=00004 IS ON CR00039 USING 00028 BLKS R=0000

FTN4,L
$EMA(CMBLK,0)
C ************************
        PROGRAM RADMI
C ************************
C
C                                       ====================
C                                        ROBERT K. SCHNEIDER
C                                       ====================
C
C * THIS PROGRAM EVALUATES THE EIGENVALUE SOLTION FOR THE RADIATED FIELD
C     OUTSIDE AN INFINITELY LONG DIELECTRIC CYLINDRICAL SHELL OF INNER
C     RADIUS B AND OUTER RADIUS C.  THE INCIDENT FIELD HAS AS ITS SOURCE
C     AN INFINITE LINE CURRENT CLOSE TO, BUT EXTERNAL TO, THE CYLINDER
C     AND AT AN ANGLE PHI-PRIME EQUAL TO ZERO.
C
C * A MATRIX EIGENVALUE SOLUTION HAS BEEN FORMULATED LEAVING THE COEFFI-
C     CIENTS OF THE FIELD TO BE SOLVED FOR.  AN ALGORITHM BY PRESCOTT D.
C     CROUT IS USED TO SOLVE FOR THESE COEFFICIENTS.
C
C * THE FAR FIELD IS GIVEN BY :
C
C             Ezωn = -jwuFn TIMES THE HANKEL FUNCTION OF THE 2nd KIND
C                     OF ARGUEMENT K0*RAU AND ORDER n.
C
C     THE TOTAL FIELD IS A SUM ON n OF THE ABOVE.     A CONVERGENCE OF THIS
C     SUM IS DETERMINED BY THIS PROGRAM.  ALSO, SINCE THE ARGUEMENT OF THE
C     ABOVE HANKEL FUNCTION IS >> n, THE APPROPRIATE ASYMPTOTIC EXPANSION
C     IS USED.
C
C * EPSLN IS A VALUE INPUT BY THE USER FOR USE IN DETERMINING CONVERGENCE.
C
C * K0 = WAVE NUMBER IN FREE SPACE
C
C * K2 = WAVE NUMBER IN DIELECTRIC
C
      INTEGER ESTIME
C
      REAL K0,K2,MAX,NRMFLD,NPHI,J0
C
      COMPLEX HN20A,HNP20A,SUMEZ,A(6,7),F(6),FN(1001),D11,D12,
     1        SUMN,SUMP,SUM,EZON,D5
C
      DIMENSION IBUFR(16),J2(10)
C
      COMMON /CMBLK/ A,F,ECHWPW(3600),DGRENC,SCRPEZ(3600),
     1               DBPRPT(3600)
C
      DATA PI/3.14159265/
      DATA XCR/4.875/,YCR/3.875/,RDS/2.748/
C
C * SET UP CALL FOR INPUT FROM DISK FILE "DATA1"
C
      DATA IBUFR/2*0,2HDA,2HTA,2H1 ,3*0,2216,7*0/
C
      CALL SFOPN(IBUFR,ISLU)
C
      CALL EXEC(22,1)
```

```
C
      READ(ISLO,*) AA,B,C,CURENT,R,EPSLN,FREQ,PERM,DOPENI,EPSLNR,
     1               ESTIME,N,ISKIP
C
C * SET DEFAULT FOR ESTIME
C
        IF(ESTIME .EQ. 0) ESTIME=500
C
      ZERO=0.0
C
C * CALCULATE THE WAVE NUMBERS
C
        K0=2.0*PI*FREQ/3.E8
        K2=K0*SQRT(EPSLNR)
C
C * COMPUTE ARGUEMENTS FOR BESEL FUNCTION CALLS
C
        X1=K0*AA
        X2=K0*B
        X3=K0*C
        X4=K2*B
        X5=K2*C
C
C * LOOP ESTIME TIMES TO DETERMINE THE EIGENVALUE Fn FOR EACH ORDER n.
C
        K=0
C
      DO 100 L=-ESTIME,ESTIME
         K=K+1
         EPSLNN=1.0
C        LM1=L-1
         LM1=L
C
           CALL BESEL(LM1,X1,BB,Y,BP,YP)
          HN20A=CMPLX(BB,-Y)
         HNP20A=CMPLX(BP,-YP)
        A(1,1)=CMPLX(BB,ZERO)
        A(1,2)=CMPLX(Y,ZERO)
        A(1,3)=(0.,0.)
        A(1,4)=(0.,0.)
        A(1,5)=(0.,0.)
        A(1,6)=-HN20A
        A(1,7)=(0.,0.)
CC
        A(6,1)=CMPLX(BP,ZERO)
        A(6,2)=CMPLX(YP,ZERO)
        A(6,3)=(0.,0.)
        A(6,4)=(0.,0.)
        A(6,5)=(0.,0.)
        A(6,6)=-HNP20A
           IF(LM1 .EQ. 0) EPSLNN=0.5
          EQLMT=(CURENT*EPSLNN)/(PI*AA*K0)
        A(6,7)=CMPLX(EQLMT,ZERO)
C
           CALL BESEL(LM1,X2,BB,Y,BP,YP)
          DUM1=-K0*BP
        A(3,5)=CMPLX(-BB,ZERO)
CC
        A(5,5)=CMPLX(DUM1,ZERO)
C
```

```
          CALL BESEL(LM1,X3,BB,Y,BP,YP)
         DUM1=K0*BP
         DUM2=K0*YP
        A(2,1)=CMPLX(BB,ZERO)
        A(2,2)=CMPLX(Y,ZERO)
CC
        A(4,1)=CMPLX(DUM1,ZERO)
        A(4,2)=CMPLX(DUM2,ZERO)
C
          CALL BESEL(LM1,X4,BB,Y,BP,YP)
         DUM1=K2*BP
         DUM2=K2*YP
        A(3,1)=(0.,0.)
        A(3,2)=(0.,0.)
        A(3,3)=CMPLX(BB,ZERO)
        A(3,4)=CMPLX(Y,ZERO)
        A(3,6)=(0.,0.)
        A(3,7)=(0.,0.)
CC
        A(5,1)=(0.,0.)
        A(5,2)=(0..0.)
        A(5,3)=CMPLX(DUM1,ZERO)
        A(5,4)=CMPLX(DUM2,ZERO)
        A(5,6)=(0.,0.)
        A(5,7)=(0.,0.)
C
          CALL BESEL(LM1,X5,BB,Y,BP,YP)
         DUM1=-K2*BP
         DUM2=-K2*YP
        A(2,3)=CMPLX(-BB,ZERO)
        A(2,4)=CMPLX(-Y,ZERO)
        A(2,5)=(0.,0.)
        A(2,6)=(0.,0.)
        A(2,7)=(0.,0.)
CC
        A(4,3)=CMPLX(DUM1,ZERO)
        A(4,4)=CMPLX(DUM2,ZERO)
        A(4,5)=(0.,0.)
        A(4,6)=(0.,0.)
        A(4,7)=(0.,0.)
C
C * NOW THAT THE MATRIX A IS FORMED, CALL CLSKY TO SOLVE FOR THE
C    EIGENVALUES.
C
         M=N+1
        CALL CLSKY(N,M)
C
C * PULL OUT Fn FROM ARRAY F
C
         IF(K .NE. 10) GO TO 17
C       DO 19 I=1,6
C         WRITE(6,18) F(I),L,I
18       FORMAT(1X,"F= ",1E15.8,2X,E15.8,5X,I4,5X,I4,/)
C19      CONTINUE
C        WRITE(6,70)
         K=0
C
17      FN(L+ESTIME+1)=F(6)
C
C        DO 60 M=1,6
```

```
C             WRITE(6,50) F(M),M,L
50           FORMAT(1X,"F=",2X,1E15.8,2X,E15.8,5X,I4,5X,I4)
C60           CONTINUE
C             WRITE(6,70)
70           FORMAT(1X,//)
C
100       CONTINUE
C
          WRITE(1,200)
200       FORMAT(1X,"FN EIGENVALUES CALCULATED.")
C
C * CALCULATE THE ECHO WIDTH PER WAVELENGTH RELATIVE TO THE INCIDENT
C    FIELD AT THE CENTER OF THE OBSTACLE
C
            SFLDMX=0.0
            TFLDMX=0.0
            IAVNTN=0.0
            IMAX=0
          NTIMES=IFIX(360./DGRENC)
          DO 700 I=1,NTIMES
             IM1=I-1
             PHI=IM1*DGRENC*PI/180.
             NCNVRG=0
             SUMEZ=(0.,0.)
              SUMM=(0.,0.)
              SUMP=(0.,0.)
C
          DO 550 J=1,ESTIME
             JM1=J-1
             ARGP=JM1*PI/2.
1           D1=SIN(ARGP)
2           D2=COS(ARGP)
              IF(JM1 .EQ. 0) GO TO 540
             ARGM=-ARGP
3           D3=SIN(ARGM)
4           D4=COS(ARGM)
5          SUMM=FN(JM1)*COS(-JM1*PHI)*CMPLX(D3,-D4)
540        SUMP=FN(JM1+ESTIME+1)*COS(JM1*PHI)*CMPLX(D1,-D2)
C
          SUMEZ=SUMEZ+2.*PI*FREQ*PERM*(SUMM+SUMP)
C
6          D10=D2*AIMAG(FN(JM1+ESTIME+1))*COS(JM1*PHI)*2.*PI*FREQ*PERM
               IF(D10 .GT. EPSLN) GO TO 550
                NCNVRG=NCNVRG+1
                J2(NCNVRG)=J
               IF(NCNVRG .LT. 10) GO TO 550
                DO 425 L=2,10
                  IF(J2(L) .NE. J2(L-1)+1) GO TO 427
425             CONTINUE
                 GO TO 510
427            NCNVRG=10-L
                DO 426 L=1,10
                  IF(L .GT. NCNVRG) GO TO 428
                 J2(L)=J2(10-NCNVRG+L)
                  GO TO 426
428              J2(L)=0
426             CONTINUE
550            CONTINUE
C
                WRITE(6,506)
```

```
506           FORMAT(1X,"TRIED MORE THAN ESTIME TERMS IN SUM.",/)
C
C510            WRITE(6,520) I,J
520           FORMAT(1X,"I=",2X,I4,5X,"J=",2X,I4,/)
C
510             IF(J .GT. IMAX) IMAX=J
              IAVNIM=IAVNIM+J
               IF(ISKIP .NE. 1) GO TO 530
C
             ARGCS=((K0*R)-(PI/4.0))/(200*PI)
              GO TO 7
530          ARGCS=(K0*R)-(PI/4.0)
C
C            WRITE(6,79) ARGCS
79         FORMAT(1X,"ARGCS= ",E15.8,/)
C
7           D1=COS(ARGCS)
C
C             WRITE(6,77)
77          FORMAT(1X,"GETTING HERE 7",/)
C
8           D2=-SIN(ARGCS)
C
C             WRITE(6,78)
78          FORMAT(1X,"GETTING HERE 8",/)
C
            D3=SQRT(2/(PI*K0*R))
            D5=D3*CMPLX(D1,D2)
          EZ0N=D5*SUMEZ
C
          SCRPEZ(I)=CABS(EZ0N)
C
            IF(SCRPEZ(I) .GT. TFLDMX) TFLDMX=SCRPEZ(I)
C
             ARG=K0*ABS(AA)
              CALL BESEL(0,ARG,BB,Y,BP,YP)
           * J0=BB*BB
             Y0=Y*Y
             D1=3.E8**4.*8.854E-12**2.*8./(FREQ**2.*CURENT**2.*PI**3.)
9             ARG=K0*AA*COS(PHI)
11           D2=COS(ARG)
14           D3=SIN(ARG)
             D4=2.*PI*FREQ*CURENT/(4.*3.E8**2.*8.854E-12)
             D12=D4*CMPLX(D2,D3)
             D11=SUMEZ+D12
            XMAG=CABS(D11)*CABS(D11)
C
            ECHWPW(I)=D1*XMAG/(J0+Y0)
C
C             WRITE(6,1000) ECHWPW(I),PHI
C1000          FORMAT(1X,"ECHWPW=",2X,1E15.8,5X,"PHI=",2X,F7.3,/)
C
             IF(ECHWPW(I) .GT. SFLDMX) SFLDMX=ECHWPW(I)
C
700        CONTINUE
C
                SCRPEZ(NTIMES+1)=SCRPEZ(1)
C
C            WRITE(6,1200)
1200        FORMAT(1X,////)
```

```
C
              WRITE(1,23)
23            FORMAT("PLOT THE SCATTERED FIELD?& Y/N")
              READ(1,12) IANS
               IF(IANS .EQ. 1H ) GO TO 505
               IF(IANS .NE. 1HY) GO TO 507
C
C * PLOT THE ECHO WIDTH PER WAVELNGTH VS ANGLE
C
505           CALL ECOPT(SFLDMX)
C
507           DO 900 I=1,NTIMES
                 PHI=(I-1.)*DGRENC
C               WRITE(6,800) ECHWPW(I),PHI
800            FORMAT(1X,"ECHWPW=",2X,1E15.8,5X,"PHI=",2X,F7.3,/)
900           CONTINUE
C
               WRITE(1,29)
29            FORMAT("PLOT THE TOTAL FIELD?& Y/N")
               READ(1,12) IANS
12            FORMAT(A1)
                IF(IANS .EQ. 1H ) GO TO 555
                IF(IANS .NE. 1HY) GO TO 950
C
555            WRITE(1,13)
13            FORMAT("NEW GRID?& Y/N")
               READ(1,12) IANSS
                IF(IANSS .EQ. 1H ) GO TO 560
                IF(IANSS .NE. 1HY) GO TO 565
C
560           CALL POLAR(0)
              CALL LABL(0)
C
565            WRITE(1,28)
28            FORMAT("ENTER PEN# FOR DATA.&")
               READ(1,*) IPND
               WRITE(25,22) IPND
22            FORMAT("SP",I1)
C
950           DO 600 I=1,NTIMES+1
                 PHI=((I-1)*DGRENC+180.)*PI/180.
                DBPRPT(I)=20.*ALOGT(SCRPEZ(I)/TFLDMX)
C
                  PHII=(I-1)*DGRENC
C                WRITE(6,33) DBPRPT(I),PHII
33            FORMAT(1X,"GAIN - POWER = ",E15.8," db",5X,"ANGLE = ",
     1              F7.3,/)
C
                 IF(DBPRPT(I) .LT. -40.) DBPRPT(I)=-40.
                DUMMY=(DBPRPT(I)+40.)/40.
               IX=INT((XCR-DUMMY*COS(PHI)*RDS)*1000.)
               IY=INT((YCR-DUMMY*SIN(PHI)*RDS)*1000.)
                 IF(IANS .NE. 1H .AND. IANS .NE. 1HY) GO TO 600
                WRITE(25,24) IX,IY
24             FORMAT("PA",I5",","I5";PD")
600            CONTINUE
C
                WRITE(25,26)
26             FORMAT("PU")
C
```

```
      CALL LABEL(1)
C
      DO 940 I=1,NTIMES
         PHI=(I-1)*DGRENC
       WRITE(6,935) DEPRPT(I),PHI
935    FORMAT(1X,"POWER GAIN = ",E15.8," db",5X,"ANGLE =",
     1          F7.3," DEGREES",/)
940   CONTINUE
C
      WRITE(6,70)
C950   WRITE(6,945) SFLDMX
945   FORMAT(1X,"THE ECHWFW-MAX = ",E15.8,//)
C      WRITE(6,957) TFLDMX
957   FORMAT(1X,"THE TOTAL FIELD-MAX= ",E15.8)
C      WRITE(6,70)
C
      IAVNTM=IAVNTM/NTIMES
C     WRITE(6,947)
947  FORMAT(1X,"THE AVERAGE NUMBER OF TERMS IN THE INFINITE SUM")
C     WRITE(6,948) NTIMES
948  FORMAT(1X,"FOR ",I5," NUMBER OF ANGLES BETWEEN 0 & 360 IS",/)
C     WRITE(6,949) IAVNTM
949  FORMAT(1X,I5,"  TERMS.")
C     WRITE(6,70)
C     WRITE(6,951) IMAX
951  FORMAT(1X,"AND, THE MAX NUMBER OF TERMS WAS ",I5,"  TERMS.")
C
      CALL EXEC(23,5HSMP  ,4,ISLU)
C
      END
      END$
```

```
Y T=00004 IS ON CR00039 USING 00005 BLKS R=0000

 FTN4.L
 $EMA(CMBLK,0)
 C ***********************************
          SUBROUTINE CLSKY(N,M)
 C ***********************************
 C
          COMPLEX A(6,7),F(6),
       1          SUM
 C
 C
          COMMON /CMBLK/ A,F,ECHWPW(3600),DGRENC,SCRPEZ(3600),
       1                 DBPRPT(3600)
 C
          DATA PI/3.14159265/
 C
 C * CALCULATE FIRST ROW OF UPPER UNIT TRIANGULAR MATRIX
 C
          DO 3 J=2,M
 3        A(1,J)=A(1,J)/A(1,1)
 C
 C * CALCULATE OTHER ELEMENTS OF U AND L MATRICES
 C
          DO 8 I=2,N
          J=I
          DO 5 II=J,N
          SUM=(0.0,0.0)
          JM1=J-1
          DO 4 K=1,JM1
 4        SUM=SUM+A(II,K)*A(K,J)
 5        A(II,J)=A(II,J)-SUM
          IP1=I+1
          DO 7 JJ=IP1,M
          SUM=(0.0,0.0)
          IM1=I-1
          DO 6 K=1,IM1
 6        SUM=SUM+A(I,K)*A(K,JJ)
 7        A(I,JJ)=(A(I,JJ)-SUM)/A(I,I)
 8        CONTINUE
 C
 C * SOLVE FOR F(I) BY BACK SUBSTITUTION
 C
          F(N)=A(N,N+1)
          L=N-1
          DO 10 NN=1,L
          SUM=(0.0,0.0)
          I=N-NN
          IP1=I+1
          DO 9 J=IP1,N
 9        SUM=SUM+A(I,J)*F(J)
 10       F(I)=A(I,M)-SUM
          RETURN
          END
          END$
```

```
I 1-00004 IS ON CR00039 USING 00006 BLKS R=0000

 FTN4,L
 $EMA(CMBLK,0)
 C ************************************
         SUBROUTINE ECOPT(XMAX)
 C ************************************
 C
         COMPLEX A(6,7),F(6)
 C
 C
         COMMON /CMBLK/ A,F,ECHWPW(3600),DGRENC,SCRPEZ(3600),
      1                    DBPRPT(3600)
 C
 C
         DATA PI/3.14159265/
         NX0P=2.*1000
         NY0P=1.75*1000
 C
         NXSTP=0.8*1000
         NYSTP=0.5*1000
 C
         NXSTPS=9
         NYSTPS=10
 C
         IPEN=1
         WRITE(25,3) IPEN
 C
          NXEP=NX0P+NXSTPS*NXSTP
         WRITE(25,1) NX0P,NY0P
         WRITE(25,1) NXEP,NY0P
          WRITE(25,2)
 C
          NYEP=NY0P+NYSTPS*NYSTP
         WRITE(25,1) NX0P,NY0P
         WRITE(25,1) NX0P,NYEP
          WRITE(25,2)
 C
         DO 100 J=1,NXSTPS
           IX=NX0P+J*NXSTP
           IY1=NY0P-50
           IY2=NY0P+50
          WRITE(25,1) IX,IY1
          WRITE(25,1) IX,IY2
           WRITE(25,2)
 100     CONTINUE
 C
         DO 200 I=1,NYSTPS
           IY=NY0P+I*NYSTP
           IX1=NX0P-50
           IX2=NX0P+50
          WRITE(25,1) IX1,IY
          WRITE(25,1) IX2,IY
           WRITE(25,2)
 200    CONTINUE
 C
        CALL ECLBL
        CALL LABEL(1)
 C
        NPTS=IFIX(180./DGRENC)
```

```
C
      IPEN=3
      WRITE(25,3) IPEN
C
      DO 400 IPHI=1,NPTS
         IX=IFIX((IPHI-1.)*DGRENC/20.*NXSTP+2000.)
         IY=IFIX((ECHWPW(IPHI)/XMAX)/0.1*NYSTP+1750.)
       WRITE(25,1)IX,IY
400   CONTINUE
C
       WRITE(25,2)
1     FORMAT("PA",I5",","I5,";PD")
2     FORMAT("PU")
3     FORMAT("IN;SP",I1)
       END
       END$
```

```
L T=00063 IS ON CR00033 USING 00015 BLKS R-0000

 FTN4,L
       SUBROUTINE BESEL(N,X,B,Y,BP,YP)
 C
 C     COMPUTE THE BESSEL FUNCTION OF ORDER N, N AN INTEGER N>OR=0
 C     WITH REAL ARGUMENT
 C     ALL EQUATION REFERENCES TO ABRAMOWITZ AND STEGUN
 C
       DIMENSION C0(7),C1(7),D0(7),D1(7),E0(7),E1(7),G0(7),G1(7)
       DATA C0/1.0,-2.2499997,1.2656208,-.3163866,.444479E-1,
      *-.39444E-2,.21E-3/
       DATA C1/.5,-.56249985,.21093573,-.3954289E-1,.443319E-2,
      *-.31761E-3,.1109E-4/
       DATA D0/.79788456,-.77E-6,-.55274E-2,-.9512E-4,.137237E-2,
      *-.72805E-03,.14476E-03/
       DATA D1/.79788456,.156E-5,.1659667E-1,.17105E-3,-.249511E-2,
      *.113653E-02,-.20033E-03/
       DATA E0/-.78539816,-.4166397E-1,-.3954E-4,.262573E-2,-.54125E-3,
      *-.29333E-03,.13558E-03/
       DATA E1/-2.35619449,.12499612,.565E-4,-.637879E-2,.74348E-3,
      *.79824E-03,-.29166E-03/
       DATA G0/.3674669,.6055937,-.7435038,.2530012,-.426121E-01,
      *.427916E-02,-.24846E-03/
       DATA G1/-.6366198,.2212091,2.168271,-1.316483,.312395,
      *-.400976E-01,.27873E-02/
       DATA PI/3.1415926/
       IFLG=0
       IF (N.LT.0) IFLG=1
       N=IABS(N)
       IF(ABS(X).LT.1.0E-10)GO TO 150
       IF(ABS(X).GT.3.0)GO TO 50
       X3SQ=X*X/9.0
       PROD=1.0
       B0=0.0
       B1=0.0
       CUM0=0.0
       CUM1=0.0
 C     SEE EQUATIONS 9.41 AND 9.44
       DO 5 I=1,7
       B0=B0+C0(I)*PROD
       B1=B1+C1(I)*PROD
       CUM0=CUM0+G0(I)*PROD
       CUM1=CUM1+G1(I)*PROD
       PROD=PROD*X3SQ
 5     CONTINUE
       B1=B1*X
       XC=2.0*SNGL(DLOG(DBLE(0.5*X)))/PI
       Y0=XC*B0+CUM0
       Y1=XC*B1+CUM1/X
       GO TO 100
 C     EQS 9.4.3 AND 9.4.6
 50    THROVX=3.0/X
       PROD=1.0
       F0=0.0
       F1=0.0
       THETA0=X
       THETA1=X
       DO 55 I=1,7
       F0=F0+D0(I)*PROD
```

```
      F1=F1+D1(I)*PROD
      THETA0=THETA0+E0(I)*PROD
      THETA1=THETA1+E1(I)*PROD
      PROD=PROD*THROVX
55    CONTINUE
      SQRX=1.0/SNGL(DSQRT(DBLE(X)))
      B0=SQRX*F0*SNGL(DCOS(DBLE(THETA0)))
      B1=SQRX*F1*SNGL(DCOS(DBLE(THETA1)))
      Y0=SQRX*F0*SNGL(DSIN(DBLE(THETA0)))
      Y1=SQRX*F1*SNGL(DSIN(DBLE(THETA1)))
100   IF(N-1)101,105,110
101   B=B0
      BP=-B1
      Y=Y0
      YP=-Y1
      GO TO 200
105   B=B1
      BP=B0-B1/X
      Y=Y1
      YP=Y0-Y1/X
      GO TO 200
C     FOR RECURRSIVE DIRECTION COMMENTS SEE SECTION 9.12,P385
110   XN=N
      IF(XN.LT.ABS(X))GO TO 130
C     FOR X< N RECUR DOWNWARD
      BLAST=1.0
      BLASTP=0.0
      J=N+10
      DO 115 I=1,J
      XI=J-I
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST =BNEXT
      IF(I.NE.10)GO TO 115
      BLP=BLASTP
      Z=BNEXT
115   CONTINUE
      IF(ABS(B0).LT.ABS(B1))GO TO 117
      CORR=B0/BLASTP
      GO TO 118
117   CORR=-B1/BNEXT
118   B=BLP*CORR
      BNMIN1=Z*CORR
      GO TO 140
C     FOR X>N RECUR UPWARD
130   BLASTP=B0
      BLAST=B1
      DO 135 I=2,N
      XI=I-1
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST=BNEXT
135   CONTINUE
      B=BLAST
      BNMIN1=BLASTP
140   BP=BNMIN1-XN*B/X
      YLASTP=Y0
      YLAST=Y1
      DO 145 I=2,N
      XI=I-1
```

```
      YNEXT=2.0*XI*YLAST/X-YLASTP
      YLASTP=YLAST
      YLAST=YNEXT
145   CONTINUE
      Y=YLAST
      YP=YLASTP-XN*Y/X
      GO TO 200
150   B=0.0
      BP=0.0
      Y=0.0
      YP=0.0
      IF(N-1)155,160,200
155   B=1.0
      GO TO 200
160   BP=0.5
      B=0.5*X
200   IF (IFLG.EQ.0) RETURN
      B=(-1)**N*B
      N=-N
      RETURN
      END
      END$
```

```
,P T=00004 IS ON CR00039 USING 00005 BLKS R=0000

C       I. MAIN Program
C
C          1) Reads inputs
C
C          2) Calls CELL
C             a) AN
C             b) NCELLS
C
C          3) Calls CLCRD
C             a) XN
C             b) YN
C
C          4) Calls ECHOW
C             a) W(PHI)/WAVELENGTH
C
C      II. ECHOW (need Ei and En)
C
C          1) Calls FDLTL
C             a) En
C
C          2) Calls FLDNC
C             a) Ei as a function of PHI
C
C     III. FLDTL (need Emi)
C
C          1) Calls FLDNC with PTOBS = 0
C             a) Emi
C
C          2) Sets up matrix and calls CLSKY
C             a) En
C
C >>> Have En and Ei >>> Have ECHWPW
C
C      IV. MAIN
C
C          5) Calls FLDCT
C             a) Es(rauo,phi) <<< Knowing En which is in an array in
C                 EMA calculated in II.1
C
```

```
!M T=00004 IS ON CR00039 USING 00020 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
 C *******************************************
            PROGRAM RADMM
 C *******************************************
 C
 C                        ==================
 C                        ROBERT K. SCHNEIDER
 C                        ==================
 C
 C * This program uses the MOMENT METHOD as presented by JACK RICHMOND
 C   ("Scattering by a Dielectric Cylinder of Arbitrary Cross Section
 C   Shape", IEEE APS, Nov 1964, p. 334) to determine the Scattered
 C   Field from and Echo Width of a dielectric cylindrical shell of
 C   circular cross section.
 C
 C * The dimensions of the scatterer are read from a data file - DATAFL
 C
       INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
 C
       REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
      1     ECHWPW
 C
       COMPLEX CMN,EINC1,EINC0,EINC2,
      1        ESCAT,
      2        TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
      3        TAU,VIN,VOUT,ONE,ZERO,TLFLD,A0,A1
 C
       DIMENSION IBUFR(16)
 C
       COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
      1                EINC1(360),VIN(1851),ESCAT(360),
      2                ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
      3  *             EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
 C
       EQUIVALENCE (NCELLS,NZ)
 C
       DATA PI/3.14159265/
       DATA XCR/4.875/,YCR/3.875/,RDS/2.748/
       DATA IBUFR/2*0,2HDA,2HTA,2HFL,3*0,221B,7*0/
 C
 C * SET SPOOL FOR INPUT FROM IBUFR THROUGH ISLU
 C
       CALL SPOPN(IBUFR,ISLU)
       CALL EXEC(22,1)
 C
       READ(ISLU,*) AA,B,C,CURENT,R,FREQ,PERM,DGRENC,EPSLNR,NCELLS,
      1             HLFCEL
 C
 C * DETERMINE THE CELL STRUCTURE AND # OF CELLS
 C
        W=0.0
       CALL CELL(W)
 C****
          IF(HLFCEL .NE. 1) GO TO 50
 C
        NCELLS=NCELLS/2
 C****
```

```
C
C * DETERMINE THE COORDINATES OF THE CENTERS OF EACH CELL
C
C50      CALL CLCRD(W)
C
C * DETERMINE THE ECHO WIDTH PER WAVELENGTH
C
50       XMAX=0.
        CALL ECHOW(XMAX,W)
C
         NPTS=IFIX(360./DGRENC)
C
           WRITE(1,23)
23        FORMAT("PLOT THE SCATTERED FIELD?& Y/N")
           READ(1,12) IANS
12        FORMAT(A1)
            IF(IANS .EQ. 1H ) GO TO 210
            IF(IANS .NE. 1HY) GO TO 220
C
C * PLOT ECHO WIDTH PER WAVELENGHT
C
210     CALL ECOPT(XMAX)
C
C * DETERMINE THE FAR ZONE SCATTERED FIELD AT EACH DGRENC
C
220     CALL FLDCT
C
C * DETERMINE THE INCIDENT FIELD IN THE FAR ZONE
C
        CALL FLDNC(1,W)
C
        TFLDMX=0.0
C
        DO 260 I=1,NPTS
          TLFLD=EINC1(I)+ESCAT(I)
         SCRPEZ(I)=CABS(TLFLD)
         IF(SCRPEZ(I) .GT. TFLDMX) TFLDMX=SCRPEZ(I)
260     CONTINUE
C
          SCRPEZ(NPTS+1)=SCRPEZ(1)
C
         WRITE(1,24)
24      FORMAT("PLOT THE TOTAL FIELD?& Y/N")
        READ(1,12) IANS
         IF(IANS .EQ. 1H ) GO TO 230
         IF(IANS .NE. 1HY) GO TO 270
C
230      WRITE(1,25)
25      FORMAT("NEW GRID?& Y/N")
        READ(1,12) IANS
         IF(IANS .EQ. 1H ) GO TO 240
         IF(IANS .NE. 1HY) GO TO 250
C
240     CALL POLAR(0)
        CALL LABL(0)
C
250      WRITE(1,28)
28      FORMAT("ENTER PEN# FOR DATA.& ")
        READ(1,*) IPND
         WRITE(25,22) IPND
```

```
22      FORMAT("SP",I1)
C
        DO 700 I=1,NPTS+1
         PHI=((I-1)*DGRENC+180.)*PI/180.
         DBPRPT(I)=20.*ALOGT(SCRPEZ(I)/TFLDMX)
          IF(DBPRPT(I) .LT. -40.) DBPRPT(I)=-40.
          DUMMY=(DBPRPT(I)+40.)/40.
         IX=INT((XCR-DUMMY*COS(PHI)*RDS)*1000.)
         IY=INT((YCR-DUMMY*SIN(PHI)*RDS)*1000.)
          WRITE(25,29) IX,IY
29       FORMAT("PA",I5",","I5";PD")
700      CONTINUE
          WRITE(25,31)
31       FORMAT("PU")
C
270       WRITE(1,26)
26       FORMAT("HARD COPY OF CALCULATIONS?& Y/N")
         READ(1,12) IANS
          IF(IANS .EQ. 1H ) GO TO 800
          IF(IANS .NE. 1HY) GO TO 500
C
800       WRITE(6,850) TFLDMX
850      FORMAT(1X,"TOTAL FIELD-MAX = ",E15.8,//)
C
         DO 870 I=1,NPTS
           PHI=(I-1)*DGRENC
           WRITE(6,860) PHI,ECHWPW(I),DBPRPT(I)
860       FORMAT(1X,"ANGLE= ",F5.2,5X,"ECHWPW= ",E15.8,5X,
     1           "DB GAIN= ",E15.8,/)
870      CONTINUE
C
          WRITE(6,27)
27       FORMAT(1X,///)
C
         DO 400 I=1,NPTS
            PHI=(I-1)*DGRENC
            XMAG=CABS((ESCAT(I)))
           WRITE(6,300) XMAG,PHI
300       FORMAT(1X,"MAGNITUDE ESCAT=",2X,1E15.8,5X,"PHI=",2X,F7.3,/)
400      CONTINUE
C
500     CALL EXEC(23,5HSMP  ,4,ISLU)
         END
         END$
```

```
T=00004 IS ON CR00032 USING 00004 BLKS R=0000

FTN4,L
$EMA(BLKMM,0)
C ***************************************************
      SUBROUTINE CELL(W)
C ***************************************************
C
C * THIS SUBROUTINE DETERMINES THE # OF CELLS NEEDED AND THE
C     DIMENSION OF A CELL TO SPAN THE OBSTACLE.
C
C
      INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
      REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1         ECHWPW
C
      COMPLEX CMN,EINC1,EINC0,EINC2,
     1            ESCAT,
     2            TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3            TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
      DIMENSION IBUFR(16)
C
      COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                EINC1(360),VIN(1851),ESCAT(360),
     2                ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3                EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
C
      EQUIVALENCE (NCELLS,NZ)
C
C
      DATA PI/3.14159265/
C
C * THE # OF CELLS FOR THE CIRCULAR DIELECTRIC SHELL SCATTERER IS A
C     PARAMETER WHICH IS ASSUMED AT THE INITIATION OF THE PROGRAM AND
C     IS READ FROM THE DATA FILE
C
C * THE WIDTH OF EACH CELL MUST BE LESS THAN OR EQUAL TO
C     0.2 * WAVELENGTH / SQRT ( EPSLNR )
C
C * SINCE THE NUMBER OF CELLS IS A KNOWN VALUE AND THE SIZE OF THE
C     STRUCTURE IS DEFINED, THE WIDTH OF A 'SQUARE CELL', W, IS EASILY
C     DETERMINED TO BE
C
          W=2.*PI*C/NCELLS
C
C * RADIUS OF CIRCULAR CELL WITH EQUAL AREA AS 'SQUARE CELL', AN, IS
C
          ROUT=C
          RIN=C-W
        AN=SQRT((ROUT**2.-RIN**2.)/NCELLS)
C
C200     WRITE(6,100) W,AN,NCELLS
100   FORMAT(1X,"W=",2X,1E15.9,5X,"AN=",2X,1E15.9,5X,"NCELLS=",
     1         2X,I4,//)
C
      RETURN
       END
       END$
```

```
RD T=00004 IS ON CR00029 USING 00004 BLKS R=0000

  FTN4,L
  $EMA(BLKMM,0)
  C ****************************************
          SUBROUTINE CLCRD(W)
  C ****************************************
  C
  C * THIS SUBROUTINE CALCULATES THE COORDINATES OF THE CENTER OF
  C     EACH CELL.
  C
  C
          INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
  C
          REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
       1       ECHWFW
  C
          COMPLEX CMN,EINC1,EINC0,EINC2,
       1          ESCAT,
       2          TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
       3          TAU,VIN,VOUT,ONE,ZERO,A0,A1
  C
          DIMENSION IBUFR(16)
  C
          COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
       1                 EINC1(360),VIN(3702),ESCAT(360),XNN(3702),
       2                 YNN(3702),ECHWPW(360),K0,K2,VOUT(3702),TAU(3702),
       3                 EINC2,SCRPEZ(361),DBPRPT(361),A0(3702),A1(3702)
  C
          EQUIVALENCE (NCELLS,NZ)
  C
  C
  C
          DATA PI/3.14159265/
  C
  C * RADIUS OUT TO CENTER OF CELL n
  C
          RN=C-(W/2.)
  C
  C * INCREMENT ANGLE FOR CELL LOCATIONS
  C
          THETAN=2.*PI/NCELLS
  C
  C        WRITE(6,50) THETAN
  50      FORMAT(1X,"THETAN=",2X,1E15.9,//)
  C
  C * DETERMINE THE CORDINATES OF THE CENTER OF EACH CELL n
  C
          DO 10 N=1,NCELLS
            THETA=(N-1)*THETAN
           XNN(N)=RN*COS(THETA)
           YNN(N)=RN*SIN(THETA)
  C
  C         WRITE(6,100) XNN(N),YNN(N),N
  100      FORMAT(1X,"XNN=",2X,1E15.9,5X,"YNN=",2X,1E15.9,5X,"N=",2X,I4,/)
  10      CONTINUE
  C
          RETURN
           END
           END$
```

```
W T=00004 IS ON CR00039 USING 00008 BLKS R=0000

 FTN4,L
 $EMA(BLKMN,0)
C ****************************************
      SUBROUTINE ECHOW(XMAX,W)
C ****************************************
C
C * THIS SUBROUTINE CALCULATES THE ECHO WIDTH PER UNIT WAVELENGHT.
C     RELATIVE TO THE INCIDENT FIELD AT THE CENTER OF THE OBSTACLE,
C     FROM A DIELECTRIC CYLINDRICAL SHELL OF CIRCULAR CROSS SECTION
C     IN THE PRESENCE OF A RADIATING CURRNT FILAMENT.
C
C
C
      INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
      REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1     ECHWPW
C
      COMPLEX CNN,EINC1,EINC0,EINC2,
     1        ESCAT,
     2        TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3        TAU,VIN,VOUT,ONE,ZERO,DUMMY4,A0,A1
C
      DIMENSION IBUFR(16)
C
      COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                EINC1(360),VIN(1851),ESCAT(360),
     2                ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3                EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
C
      EQUIVALENCE (NCELLS,N2)
C
C
      DATA PI/3.14159265/
C
C * CALL FLDTL TO DETERMINE THE TOTAL FIELD IN THE OBSTACLE
C     DUE TO THE INCIDENT FIELD UPON THE OBSTACLE
C
      CALL FLDTL(W)
C
C * CALL FLDNC TO DETERMINE THE INCIDENT FIELD AT THE CENTER OF
C     THE OBSTACLE.
C
      CALL FLDNC(2,W)
C
      NPTS=IFIX(360./DGRENC)
C
C       DO 7 I=1,NPTS
C         WRITE(6,5) EINC2
5       FORMAT(1X,"ECHOW : EINC2=",2X,1E15.8,2X,E15.8,/)
C7      CONTINUE
C
C * DETERMINE THE MAGNITUDE SQUARED OF THE INCIDENT FIELD
C
       EMAGI=CABS((EINC2))*CABS((EINC2))
C
      DUMMY1=K0*PI**2.*FREQ/(3.E8*EMAGI)
C
```

```
C * LOOP NCELLS TIMES FOR THE "SCATTERED FIELD" IN THE FAR ZONE.
C
         DO 20 IPHI=1,NPTS
            PHI=(IPHI-1.)*DGRENC*PI/180.
C
          DUMMY4=(0.,0.)
C
         RN=C-(W/2.)
         THETAN=2.*PI/NCELLS
C
         DO 10 N=1,NCELLS
            THETA=(N-1)*THETAN
             XNN=RN*COS(THETA)
             YNN=RN*SIN(THETA)
           ARGJ1=K0*AN
            CALL BESEL(1,ARGJ1,BB,Y,BP,YP)
           J1=BB
           ARGH=K0*(XNN*COS(PHI)+YNN*SIN(PHI))
           DUMMY2=COS(ARGH)
           DUMMY3=SIN(ARGH)
          DUMMY4=DUMMY4+(EPSLNR-1.)*VOUT(N)*AN*J1*CMPLX(DUMMY2,DUMMY3)
10       CONTINUE
C
          EMAGN=CABS(DUMMY4)*CABS(DUMMY4)
C
         ECHWPW(IPHI)=DUMMY1*EMAGN
CC
          IF(ECHWPW(IPHI) .GT. XMAX) XMAX=ECHWPW(IPHI)
CC
C
              PHII=PHI*180./PI
C
C         WRITE(6,100) ECHWPW(IPHI),PHII
100      FORMAT(1X,"ECHWPW=",2X,1E15.9,5X,"PHI=",2X,F7.3,/)
C
20       CONTINUE
C
         RETURN
          END
          END$
```

```
L T=C0004 IS ON CR00039 USING 00008 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
 C ********************************
         SUBROUTINE FLDTL(W)
 C ********************************
 C
 C * THIS SUBROUTINE CALCULATES THE TOTAL FIELD IN THE OBSTACLE
 C
 C
         INTEGER NCELLS,NPTS,RTOBS,IPHI,MCELLS
 C
         REAL XNN,YNN,EMAG1,J1,J0,K0,K2,Y0,Y1,EMAGN,
      1       ECHWPW
 C
         COMPLEX CMN,EINC1,EINC0,EINC2,
      1         ESCAT,
      2         TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
      3         TAU,VIN,VOUT,ONE,ZERO,A0,A1
 C
         DIMENSION IBUFR(16)
 C
         COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
      1                 EINC1(360),VIN(1851),ESCAT(360),
      2                 ECHWPW(360),K0,K2,VOUT(1951),TAU(1951),
      3                 EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
 C
         EQUIVALENCE (NCELLS,N2)
 C
 C
 C
         DATA PI/3.14159265/
 C
 C * OBTAIN THE FIELD INCIDENT UPON THE OBSTACLE
 C
         CALL FLDNC(0,W)
 C
 C        DO 5 I=1,NCELLS
 C          WRITE(6,7) EINC0(I),I
 7         FORMAT(1X,"FLDTL : EINC0=",2X,1E15.8,2X,E15.8,5X,"I=",2X,
      1        I4,/)
 C5        CONTINUE
 C
          RN=C-(W/2.)
          THETAN=2.*PI/NCELLS
 C
 C       DO 30 M=1,NCELLS
 CC
               M=1
                 THETAM=(M-1)*THETAN
                XNM=RN*COS(THETAM)
                YNM=RN*SIN(THETAM)
 CC
 C
         DO 20 N=1,NCELLS
 C
 C * DETERMINE THE WEIGHT Cmn ON En
 C
              THETA=(N-1)*THETAN
```

```
              XNN=RN*COS(THETA)
              YNN=RN*SIN(THETA)
            IF(N .NE. 1) GO TO 10
C
             DUMMY1=K0*AN
              CALL BESEL(1,DUMMY1,BB,Y,BP,YP)
             J1=BB
             Y1=Y
C
C             WRITE(6,8) J1,Y1,BB
8            FORMAT(1X,"FLDTL : J1=",2X,1E15.8,5X,"Y1=",2X,1E15.8,5X,
     1             "BB=",2X,1E15.8,/)
C
            CMN=1.+(EPSLNR-1.)*((PI*K0*AN/2.)*(CMPLX(Y1,J1))+1.)
C           A(M,N)=CMN
               IF(M .NE. 1) GO TO 20
            TAU(N)=CMN
             GO TO 20
C
10           DUMMY1=PI*K0*AN/2.
             RMN=SQRT((XNM-XNN)*(XNM-XNN) +
     1              (YNM-YNN)*(YNM-YNN))
             ARGH=K0*RMN
             ARGJ=K0*AN
              CALL BESEL(1,ARGJ,BB,Y,BP,YP)
             J1=BB
              CALL BESEL(0,ARGH,BB,Y,BP,YP)
             J0=BB
             Y0=Y
C
            CMN=DUMMY1*(EPSLNR-1.)*J1*CMPLX(Y0,J0)
C             IF(N .NE. 1) GO TO 20
            TAU(N)=CMN
C
C           A(M,N)=CMN
20         CONTINUE
C           A(M,N+1)=-EINC0(M)
C30         CONTINUE
C
C * NOW THAT THE MATRIX HAS BEEN FORMED, SOLVE FOR En
C
           MCELLS=NCELLS+1
C             WRITE(6,250) MCELLS,NCELLS
250       FORMAT(1X,"MCELLS=",2X,I4,5X,"NCELLS=",2X,I4,//)
C          DO 400 M=1,NCELLS
C
C          DO 300 N=1,MCELLS
C             WRITE(6,350) A(M,N),M,N
350       FORMAT(1X,"FLDTL : A=",2X,1E15.9,2X,E15.9,5X,"M=",2X,I4,
     1          5X,"N=",2X,I4,/)
C300        CONTINUE
C            WRITE(6,375)
375         FORMAT(1X,///)
C400        CONTINUE
C
C          DO 600 I=1,NCELLS
C             WRITE(6,700) TAU(I),I
700         FORMAT(1X,"TAU=",2X,1E15.9,2X,E15.9,5X,I4,/)
C600        CONTINUE
C
```

```
        CALL TPLD(1,XNORM,IER)
C
         WRITE(1,500) IER
500     FORMAT(1X,"IER=",2X,I4)
C
C * En ARE CONTAINED IN THE ARRAY VOUT OF DIMENSION NCELLS
C
C            DO 100 I=1,NCELLS
C             WRITE(6,200) VOUT(I),I
200          FORMAT(1X,"FLDTL : VOUT==",2X,1E15.9,2X,E15.9,5X,"I=",
     1               2X,I4,/)
C100          CONTINUE
C
        RETURN
         END
         END$
```

```
  T=00004 IS ON CR00039 USING 00009 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
C ******************************
          SUBROUTINE FLDNC(PTOBS,W)
C ******************************
C
C * THIS SUBROUTINE CALCULATES THE INCIDENT FIELD ON THE OBSTACLE,
C     AT THE FAR.ZONE POINT, AND/OR AT THE CENTER OF THE OBSTACLE.
C
C
        INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
        REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1        ECHWPW
C
        COMPLEX CMN,EINC1,EINC0,EINC2,
     1          ESCAT,
     2          TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3          TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
        DIMENSION IBUFR(16)
C
        COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1                 EINC1(360),VIN(1851),ESCAT(360),
     2                 ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3                 EINC2,SCRPED(361),DEFRPT(361),A0(1851),A1(1851)
C
        EQUIVALENCE (NCELLS,NZ)
C
C
C
        DATA PI/3.14159265/
C
        CURENT=1.0
C
        NPTS=IFIX(360./DGRENC)
C
C * WAVE # IN FREE SPACE AND IN THE OBSTACLE
C
        K0=2.*PI*FREQ/3.E8
        K2=K0*SQRT(EPSLNR)
C
C * EPSLN0 = 8.854E-12
C
         RN=C-(W/2.)
         THETAN=2.*PI/NCELLS
C
        DUMMY1=-(K0**2./(4.*2.*PI*FREQ*8.854E-12))*CURENT
C
C * IF OBSERVATION POINT AT PARTICULAR CELL, PTOBS = 0.  IF OBSER-
C     VATION POINT IN FAR ZONE AT SOME ANGLE PHI, PTOBS = 1.  IF
C     OBSERVATION POINT AT CENTER OF OBSTACLE, PTOBS = 2.
C
        IF(PTOBS .EQ. 1) GO TO 20
C
        IF(PTOBS .EQ. 2) GO TO 50
C
C * DETERMINE THE INCIDENT FIELD ON THE OBSTACLE AT EACH CELL
```

```
C     LOCATION, (Xn,Yn)
C
C * LOOP NCELLS TIMES FOR INCIDENT FIELD
C
         DO 10 I=1,NCELLS
              THETA=(I-1)*THETAN
            XNN=RN*COS(THETA)
            YNN=RN*SIN(THETA)
           DUMMY0=(YNN-0.)*(YNN-0.)
          ARGH=K0*SQRT((XNN-AA)*(XNN-AA)+DUMMY0)
           CALL BESEL(0,ARGH,BB,Y,BP,YP)
          J0=BB
          Y0=Y
C
C          WRITE(6,5) J0,Y0,BB
5         FORMAT(1X,"FLDNC : J0=",2X,1E15.8,5X,"Y0=",2X,1E15.8,5X,
     1           "BB=",2X,1E15.8,/)
C
          EINC0=DUMMY1*CMPLX(J0,-Y0)
          VIN(I)=EINC0
C
C          WRITE(6,100) EINC0,I,VIN(I)
100       FORMAT(1X,"EINC0=",2X,1E15.9,2X,E15.9,5X,"I=",I4,
     1           5X,"VIN=",2X,1E15.9,2X,E15.9,/)
10       CONTINUE
          GO TO 40
C
C * DETERMINE THE INCIDENT FIELD AT THE FAR ZONE POINT DUE TO
C     THE CURRENT FILAMENT AT RAU PRIME.  USE LARGE ARGUEMENT
C     ASYMPTOTIC EXPANSION FOR THE HANKEL FUNCTION.
C
20       DO 30 J=1,NPTS
           PHI=(J-1)*DGRENC*PI/180.
            ARGH=K0*(R-AA*COS(PHI))-PI/4.
          D1=COS(ARGH)
          D2=-SIN(ARGH)
          D3=SQRT(2./(K0*PI*R))
C
          EINC1(J)=DUMMY1*D3*CMPLX(D1,D2)
C          WRITE(6,200) EINC1(J),J
200       FORMAT(1X,"FLDNC : EINC1=",2X,1E15.8,2X,E15.8,5X,"I=",2X,
     1           I4,/)
30       CONTINUE
C
          GO TO 40
C
C * DETERMINE THE FIELD INCIDENT AT THE CENTER OF THE OBSTACLE.
C
50         ARGH=K0*ABS(AA)
            CALL BESEL(0,ARGH,BB,Y,BP,YP)
          J0=BB
          Y0=Y
C
          EINC2=DUMMY1*CMPLX(J0,-Y0)
C
40       RETURN
          END
          END#
```

```
T T=00004 IS ON CR00039 USING 00010 BLKS R=0000

 FTN4,L
 $EMA(BLKMM,0)
 C ***************************************
         SUBROUTINE ECOPT(XMAX)
 C ***************************************
 C
 C * THIS SUBROUTINE PLOTS THE NORMALIZED ECHO WIDTH PER WAVELENGTH
 C     VS. ANGLE PHI ON A LINEAR PLOT.
 C
 C
         INTEGER NCELLS,NPTS,PTOBS,IPHI,NCELLS
 C
         REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
      1      ECHWPW
 C
         COMPLEX CMN,EINC1,EINC0,EINC2,
      1         ESCAT,
      2         TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
      3         TAU,VIN,VOUT,ONE,ZERO,A0,A1
 C
         DIMENSION IBUFR(16)
 C
         COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DCRENC,EPSLNK,AN,NCELLS,
      1                 EINC1(360),VIN(1851),ESCAT(360),
      2                 ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
      3                 EINC2,SCRPE2(361),DBFRPT(361),A0(1851),A1(1851)
 C
         EQUIVALENCE (NCELLS,NZ)
 C
 C
         DATA PI/3.14159265/
         NX0P=2.*1000
         NY0P=1.75*1000
 C
         NXSTP=0.8*1000
         NYSTP=0.5*1000
 C
         NXSTPS=9
         NYSTPS=10
 C
         IPEN=1
         WRITE(25,3) IPEN
 C
          NXEP=NX0P+NXSTPS*NXSTP
         WRITE(25,1) NX0P,NY0P
         WRITE(25,1) NXEP,NY0P
          WRITE(25,2)
 C
          NYEP=NY0P+NYSTPS*NYSTP
         WRITE(25,1) NX0P,NY0P
         WRITE(25,1) NX0P,NYEP
          WRITE(25,2)
 C
         DO 100 J=1,NXSTPS
           IX=NX0P+J*NXSTP
           IY1=NY0P-50
           IY2=NY0P+50
          WRITE(25,1) IX,IY1
```

```
         WRITE(25,1) IX,IY2
          WRITE(25,2)
100      CONTINUE
C
         DO 200 I=1,NYSTPS
           IY=NY0P+I*NYSTP
           IX1=NX0P-50
           IX2=NX0P+50
          WRITE(25,1) IX1,IY
          WRITE(25,1) IX2,IY
           WRITE(25,2)
200     CONTINUE
C
        CALL ECLBL
        CALL LABEL(1)
C
        NPTS=IFIX(180./DGRENC)
C
        IPEN=3
        WRITE(25,3) IPEN
C
        DO 400 IPHI=1,NPTS
           IX=IFIX((IPHI-1.)*DGRENC/20.*NXSTP+2000.)
           IY=IFIX((ECHWFW(IPHI)/XMAX)/0.1*NYSTP+1750.)
         WRITE(25,1)IX,IY
400     CONTINUE
C
         WRITE(25,2)
1       FORMAT("PA",I5",",I5,";PD")
2       FORMAT("PU")
3       FORMAT("IN;SP",I1)
         END
         END$
```

```
T T=00004 IS ON CR00039 USING 00004 BLKS R=0000

 FTN4,L
 $EMA(BLKNM,0)
C ********************************
      SUBROUTINE FLDCT
C ********************************
C
C * THIS SUBROUTINE USES THE MOMENT METHOD TO DETERMINE THE SCATTERED
C     FIELD FROM SOME DEFINED OBSTACLE
C
C
      INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
      REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1     ECHWPW
C
      COMPLEX CMN,EINC1,EINC0,EINC2,
     1        ESCAT,
     2        TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3        TAU,VIN,VOUT,ONE,ZERO,DUMMY8,A0,A1
C
      DIMENSION IBUFR(16)
C
      COMMON /BLKNM/  AA,B,C,R,FREQ,PERM,DGRENC,EPSLNR,AN,NCELLS,
     1               EINC1(360),VIN(1851),ESCAT(360),
     2               ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
     3               EINC2,SCRPEZ(361),DBPRPT(361),A0(1851),A1(1851)
C
      EQUIVALENCE (NCELLS,NZ)
C
C
C
      DATA PI/3.14159265/
C
C * THE LARGE ARGUEMENT ASYMPTOTIC EXPANSION FOR THE HANKEL FUNCTION
C.    IS USED. SEE RICHMOND -----
C
C * LOOP NPTS TIMES TO OBTAIN THE SCATTERED FIELD AT EACH DGRENC
C
      NPTS=IFIX(360./DGRENC)
C
        RN=C-(W/2.)
        THETAN=2.*PI/NCELLS
C
      DO 20 IPHI=1,NPTS
        PHI=(IPHI-1)*DGRENC*PI/180.
        DUMMY1=(K0*R)-(PI/4.)
        DUMMY2=-COS(DUMMY1)
        DUMMY3=-SIN(DUMMY1)
        DUMMY4=SQRT(PI*K0*0.5/R)
C
C * LOOP NCELLS TIMES
C
       DUMMY8=(0.,0.)
C
      DO 10 N=1,NCELLS
         THETA=(N-1)*THETAN
          XNN=RN*COS(THETA)
          YNN=RN*SIN(THETA)
```

```
       DUMMY5=K0*(XNN*COS(PHI)+YNN*SIN(PHI))
       DUMMY6=COS(DUMMY5)
       DUMMY7=SIN(DUMMY5)
       ARGJ1=K0*AN
       CALL BESEL(1,ARGJ1,BB,Y,BP,YP)
      J1=BB
      DUMMY8=DUMMY8+(EPSLNR-1.)*VOUT(N)*AN*J1*CMPLX(DUMMY6,DUMMY7)
10    CONTINUE
C
      ESCAT(IPHI)=DUMMY4*CMPLX(DUMMY3,DUMMY2)+DUMMY8
C
20    CONTINUE
C
      RETURN
       END
       END$
```

```
 T=00004 IS ON CR00039 USING 00006 BLKS R=0000

FTN4,L
$EMA(BLKMM,0)
C ****************************************************
        SUBROUTINE TPLZ(MM,XNORM,IER)
C ****************************************************
C
C ================================================
C
C * From
C
C    "Antenna Theory and Design"
C      Warren L. Stutzman and Gary A. Thiele
C       John Wiley & Sons, New York, 1981
C       Appendix G.7 pp. 579-581
C
C ================================================
C
C * PURPOSE
C      TO SOLVE A SYSTEM INVOLVING A TOEPLITZ MATRIX.  TPLZ REQUIRES
C      ONLY 5N STORAGE LOCATIONS FOR AN N BY N MATRIX.
C
C * REMARKS
C
C A toeplitz matrix has the first row equal to the first column.
C  All elements along the main diagonal are equal.  Any diagonal
C  off the main diagonal will have this same property.
C
C * DESCRIPTION OF PARAMETERS
C
C   NZ      -ORDER OF MATRIX
C   TAU     -FIRST ROW OR COLUMN OF THE TOEPLITZ MATRIX (VECTOR
C             LENGTH NZ)
C   A0,A1   -VECTORS OF LENGTH NZ NEEDED FOR SCRATCH AREA
C   VIN     -FOR THE MATRIX EQUATION (Z)(I)=(V), VIN IS V.  (Z,I,
C             AND V MAY BE THOUGHT OF AS GENERALIZED IMPEDANCES,
C             CURRENTS, AND VOLTAGES, RESPECTIVELY). V IS A NZ BY
C             NM MATRIX.
C   MM      -NUMBER OF COLUMN VECTORS ON THE RIGHT SIDE OF MATRIX
C             EQUATION (Z)(I)=(V) (USUALLY 1).
C   XNORM   -UPON RETURN THIS IS INFINITE MATRIX NORM OF INVERSE.
C   IER     -ERROR CODE WHICH IS 0 IF NO TROUBLE.
C
C
        INTEGER NCELLS,NPTS,PTOBS,IPHI,MCELLS
C
        REAL XNN,YNN,EMAGI,J1,J0,K0,K2,Y0,Y1,EMAGN,
     1      ECHWPW
C
        COMPLEX CMN,EINC1,EINC0,EINC2,
     1          ESCAT,
     2          TAU1,ALMDA,ALPHA,COEF,FAC,C1,C2,V,V1,V2,
     3          TAU,VIN,VOUT,ONE,ZERO,A0,A1
C
        DIMENSION IBUFR(16)
C
        COMMON /BLKMM/  AA,B,C,R,FREQ,PERM,DGPENC,EPSLNR,AN,NCELLS,
     1                 EINC1(360),VIN(1851),ESCAT(360),
     2                 ECHWPW(360),K0,K2,VOUT(1851),TAU(1851),
```

```
     3                 EINC2,SCRPEZ(361),DEPAPT(361).A0(1851),A1(1851)
C
        EQUIVALENCE (NCELLS,NZ)
C
C
        DATA ONE/(1E0,0E0)/, ZERO/(0E0,0E0)/
        DATA ONNE/1E0/, ZRRO/0.E0/
C
C        WRITE(6,90) NZ,MM
90      FORMAT(1X,"NZ=",2X,I4,5X,"MM=",2X,I4,/)
         DO 150 I=1,NZ
C          WRITE(6,100) TAU(I),VIN(I),I
100     FORMAT(1X,"TPLZ : TAU=",2X,1E15.8,2X,E15.8,5X,"VIN=",2X,1E15.8.
     1          2X,E15.8,5X,"I=",I4,/)
150      CONTINUE
        N=NZ-1
        IER=0
C
C * NORMALIZE INPUT MATRIX
C
         TAU1=TAU(1)
        DO 2000 II=1,N
2000     TAU(II)=TAU(II+1)/TAU1
C
C * THE FOLLOWING CALCULATES THE ITERATIVE VARIABLES TO OBTAIN
C     A0(N) AND ALMDA
C
C * NOTE :  VECTOR A0(I) HAS I ELEMENTS AND IS STORED AS A0(I,J),
C            J=1,N
C
         ALMDA=ONE-TAU(1)*TAU(1)
         A0(1)=-TAU(1)
         I=2
    1    KK=I-1
         ALPHA=ZERO
        DO 2 M=1,KK
         LL=I-M
    2   ALPHA=ALPHA+A0(M)*TAU(LL)
         ALPHA=-(ALPHA+TAU(I))
          IF(CABS(ALPHA) .EQ. 0.D0) GO TO 15
         COEF=ALPHA/ALMDA
         ALMDA=ALMDA-COEF*ALPHA
        DO 3 J=1,KK
         L=I-J
    3   A1(J)=A0(J)+COEF*A0(L)
        DO 7 J=1,KK
    7   A0(J)=A1(J)
         A0(I)=COEF
          IF(I .GE. N) GO TO 5
    4   I=I+1
         GO TO 1
C
C * THE FOLLOWING COMPUTES VALUES OF EACH ELEMENT OF THE INVERSE
C
    5   NH=(NZ+1)/2
         FAC=ALMDA*TAU1
         XNORM=ZRRO
         NP=NZ+1
        DO 51 I=1,NH
         XNM=ZRRO
```

```
            IF(I .NE. 1) GO TO 52
            A1(1)=ONE/FAC
            XNM=CABS(A1(1))
           DO 53 J=2,NZ
            A1(J)=A0(J-1)/FAC
   53      XNM=CABS(A1(J))+XNM
            GO TO 54
   52      XNM=ZERO
            JH=I-1
            C1=A0(JH)
            NNPI=NP-I
            C2=A0(NNPI)
           DO 55 JJ=1,N
             J=NP-JJ
             INPJ=NP-J
             JL=J-1
             A1(J)=A1(JL)+(C1*A0(JL)-C2*A0(INPJ))/FAC
   55      XNM=CABS(A1(J))+XNM
            A1(1)=A0(I-1)/FAC
            XNM=XNM+CABS(A1(1))
   54        IF(XNM .GT. XNORM) XNORM=XNM
C
C * MATRIX MULTIPLY
C
           DO 56 II=1,MM
             ID=(II-1)*NZ
             V=ZERO
             V1=ZERO
           DO 57 J=1,NZ
             NIDJ=ID+J
             V2=VIN(NIDJ)
             V=V+V2*A1(J)
             KNPJ=NP-J
   57      V1=V1+V2*A1(KNPJ)
            NIDI=ID+I
            VOUT(NIDI)=V
C            WRITE(6,225) VOUT(NIDI),NIDI
225         FORMAT(1X,"TPLZ : VOUT=",2X,1E15.8,2X,E15.8,5X,"NI=",2X,I4,/)
            KIDNPI=ID+NP-I
   56      VOUT(KIDNPI)=V1
C            WRITE(6,250) VOUT(KIDNPI),KIDNPI,VOUT(NIDI),NIDI
250         FORMAT(1X,"TPLZ : VOUT=",2X,1E15.8,2X,E15.8,5X,"KI=",2X,I4,
     1            3X,"VOUT=",2X,1E15.8,2X,E15.8,5X,"NI=",2X,I4,/)
C            WRITE(6,251) VOUT(KIDNPI),KIDNPI
251         FORMAT(1X,"TPLZ : VOUT=",2X,1E15.8,2X,E15.8,5X,"KI=",2X,I4,/)
C
51         CONTINUE
C
           DO 650 I=1,NCELLS
C           WRITE(6,600) VOUT(I),I
600        FORMAT(1X,"TPLZ :: VOUT=",2X,1E15.8,2X,E15.8,5X,"I=",2X,I4,/)
650        CONTINUE
C
           RETURN
C
  15        WRITE(6,700)
 700       FORMAT(1X,"ERROR HAS OCCURRED. MATRIX IS STRONGLY NONSINGULAR")
            IER=1
C
           RETURN
```

END
END$

```
IL T=00003 IS ON CR00039 USING 00015 BLKS R=0000

FTN4,L
      SUBROUTINE BESEL(N,X,B,Y,BP,YP)
C
C     COMPUTE THE BESSEL FUNCTION OF ORDER N, N AN INTEGER N>OR=0
C     WITH REAL ARGUMENT
C     ALL EQUATION REFERENCES TO ABRAMOWITZ AND STEGUN
C
      DIMENSION C0(7),C1(7),D0(7),D1(7),E0(7),E1(7),G0(7),G1(7)
      DATA C0/1.0,-2.2499997,1.2656208,-.3163866,.444479E-1,
     *-.39444E-2,.21E-3/
      DATA C1/.5,-.56249985,.21093573,-.3954289E-1,.443319E-2,
     *-.31761E-3,.1109E-4/
      DATA D0/.79788456,-.77E-6,-.55274E-2,-.9512E-4,.137237E-2,
     *-.72805E-03,.14476E-03/
      DATA D1/.79788456,.156E-5,.1659667E-1,.17105E-3,-.249511E-2.
     *.113653E-02,-.20033E-03/
      DATA E0/-.78539816,-.4166397E-1,-.3954E-4,.262573E-2,-.54125E-3,
     *-.29333E-03,.13558E-03/
      DATA E1/-2.35619449,.12499612,.565E-4,-.637879E-2,.74348E-3,
     *.79824E-03,-.29166E-03/
      DATA G0/.3674669,.6055937,-.7435038,.2530012,-.426121E-01,
     *.427916E-02,-.24846E-03/
      DATA G1/-.6366198,.2312091,2.168271,-1.316483,.312395,
     *-.400976E-01,.27873E-02/
      DATA PI/3.1415926/
      IFLG=0
      IF (N.LT.0) IFLG=1
      N=IABS(N)
      IF(ABS(X).LT.1.0E-10)GO TO 150
      IF(ABS(X).GT.3.0)GO TO 50
      X3SQ=X*X/9.0
      PROD=1.0
      B0=0.0
      B1=0.0
      CUM0=0.0
      CUM1=0.0
C     SEE EQUATIONS 9.41 AND 9.44
      DO 5 I=1,7
      B0=B0+C0(I)*PROD
      B1=B1+C1(I)*PROD
      CUM0=CUM0+G0(I)*PROD
      CUM1=CUM1+G1(I)*PROD
      PROD=PROD*X3SQ
5     CONTINUE
      B1=B1*X
      XC=2.0*SNGL(DLOG(DBLE(0.5*X)))/PI
      Y0=XC*B0+CUM0
      Y1=XC*B1+CUM1/X
      GO TO 100
C     EQS 9.4.3 AND 9.4.6
50    THROVX=3.0/X
      PROD=1.0
      F0=0.0
      F1=0.0
      THETA0=X
      THETA1=X
      DO 55 I=1,7
      F0=F0+D0(I)*PROD
```

```
      F1=F1+D1(I)*PROD
      THETA0=THETA0+E0(I)*PROD
      THETA1=THETA1+E1(I)*PROD
      PROD=PROD*THROVX
55    CONTINUE
      SQRX=1.0/SNGL(DSQRT(DBLE(X)))
      B0=SQRX*F0*SNGL(DCOS(DBLE(THETA0)))
      B1=SQRX*F1*SNGL(DCOS(DBLE(THETA1)))
      Y0=SQRX*F0*SNGL(DSIN(DBLE(THETA0)))
      Y1=SQRX*F1*SNGL(DSIN(DBLE(THETA1)))
100   IF(N-1)101,105,110
101   B=B0
      BP=-B1
      Y=Y0
      YP=-Y1
      GO TO 200
105   B=B1
      BP=B0-B1/X
      Y=Y1
      YP=Y0-Y1/X
      GO TO 200
C     FOR RECURRSIVE DIRECTION COMMENTS SEE SECTION 9.12,P385
110   XN=N
      IF(XN.LT.ABS(X))GO TO 130
C     FOR X< N RECUR DOWNWARD
      BLAST=1.0
      BLASTP=0.0
      J=N+10
      DO 115 I=1,J
      XI=J-I
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST =BNEXT
      IF(I.NE.10)GO TO 115
      BLP=BLASTP
      Z=BNEXT
115   CONTINUE
      IF(ABS(B0).LT.ABS(B1))GO TO 117
      CORR=B0/BLASTP
      GO TO 118
117   CORR=-B1/BNEXT
118   B=BLP*CORR
      BNMIN1=Z*CORR
      GO TO 140
C     FOR X>N RECUR UPWARD
130   BLASTP=B0
      BLAST=B1
      DO 135 I=2,N
      XI=I-1
      BNEXT=2.0*XI*BLAST/X-BLASTP
      BLASTP=BLAST
      BLAST=BNEXT
135   CONTINUE
      B=BLAST
      BNMIN1=BLASTP
140   BP=BNMIN1-XN*B/X
      YLASTP=Y0
      YLAST=Y1
      DO 145 I=2,N
      XI=I-1
```

```
      YNEXT=2.0*XI*YLAST/X-YLASTP
      YLASTP=YLAST
      YLAST=YNEXT
 145  CONTINUE
      Y=YLAST
      YP=YLASTP-XN*Y/X
      GO TO 200
 150  B=0.0
      BP=0.0
      Y=0.0
      YP=0.0
      IF(N-1)155,160,200
 155  B=1.0
      GO TO 200
 160  BP=0.5
      B=0.5*X
 200  IF (IFLG.EQ.0) RETURN
      B=(-1)**N*B
      N=-N
      RETURN
      END
      END$
```

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Eigenvalue Solution Method

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The problem of scattering by thin cylindrical dielectric shells of large circular cross sections is approached by two methods: (1) an infinite series of eigenfunctions, and (2) the method of moments. Numerical results are presented for shell radii of 0.3λ, 3.0λ, and 30λ, the source being an electric line current near but external to the shell. Computer programs are presented which implement these two solutions. When the scattering structure does become large limitations on numerical results are encountered due to computer memory and speed limitations. Other difficulties are also encountered in an analysis of such

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scattering problems and are presented and discussed along with recommendations to resolve such difficulties.